EXAM MLC QUESTIONS OF THE WEEK

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Week of January 28/08

A discrete whole life annuity-due of 1 per year starting at age x has an actuarial present value of 10.00

If the mortality probability is increased from q_x to $q_x + .01$ **only for age x** (all other mortality probabilities are unchanged), then the annuity would have an actuarial present value of 9.906250 .

If the discount rate is increased from d to $d + .01$ **only for age x** (all other discount rates for future years are unchanged, and no changes in the original mortality rates), then the annuity would have an actuarial present value of 9.905263 .

Find the actuarial present value of the annuity if q_x is increased to $q_x + .01$ and d is increased to $d + .01$ **only for age x** (all other discount rates for future years are unchanged, and no changes in the original mortality rates).

The solution can be found below.

Week of January 28/08 - Solution

We use the relationship $a_x = 1 + vp_x\ddot{a}_{x+1} = 1 + (1-d)p_x\ddot{a}_{x+1}$.

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For the original annuity, $10 = 1 + (1-d)p_x\ddot{a}_{x+1}$, so that $(1-d)p_x\ddot{a}_{x+1} = 9$.

If q_x is increased to $q_x + .01$, the equation becomes
$9.906250 = 1 + (1-d)(p_x - .01)\ddot{a}_{x+1}$
(the value of the annuity at age $x+1$ is unchanged since the only change took place at age x).
From this equation we get $(1-d)(p_x - .01)\ddot{a}_{x+1} = 8.906250$, and then
$\frac{(1-d)(p_x-.01)\ddot{a}_{x+1}}{(1-d)p_x\ddot{a}_{x+1}} = \frac{p_x-.01}{p_x} = \frac{8.906250}{9}$. Solving for p_x results in $p_x = .96$.

If d is increased to $d + .01$, the equation becomes
$9.905263 = 1 + (1-d-.01)p_x\ddot{a}_{x+1}$ (again the value of the annuity at age $x+1$ is unchanged since the only change took place at age x).
From this equation we get $(.99-d)p_x\ddot{a}_{x+1} = 8.905263$, and then
$\frac{(.99-d)p_x\ddot{a}_{x+1}}{(1-d)p_x\ddot{a}_{x+1}} = \frac{.99-d}{1-d} = \frac{8.905263}{9}$. Solving for d results in $d = .0500$.

From $a_x = 1 + (1-d)p_x\ddot{a}_{x+1}$, we get $\ddot{a}_{x+1} = \frac{9}{(1-d)p_x} = \frac{9}{(.95)(.96)} = 9.868421$.

If q_x is increased to $q_x + .01 = .05$ and if d is increased to $d + .01 = .06$ (only for the first year) then the new annuity value is $1 + (1-.06)(.95)(9.868421) = 9.8125$.