EXAM MLC QUESTIONS OF THE WEEK

S. Broverman, 2008

Week of January 14/08

A fully continuous whole life insurance of face amount 1 is based on the following assumptions:

- constant force of interest of 6%
- constant force of mortality of .015 at all ages
- annual contract premium of .016

An insurer determines that with a portfolio of n independent policies of this type, using the normal approximation, the probability of a positive loss on all policies combined is .05. How many additional independent policies of the same type would be needed to reduce the probability of a positive total loss to .025 (continuing to use the normal approximation)?

A) Less than 500 B) At least 500, but less than 600 C) At least 600, but less than 700
D) At least 700, but less than 800 E) At least 800

The solution can be found below.

Week of January 14/08 - Solution

The loss on one policy is $L = Z - QY = Z - .016Y$,
where Z is the present value random variable for a continuous whole life insurance of 1 and Y is the PVRV of a continuous life annuity of 1 per year.

$E[L] = \overline{A}_x - .016\overline{a}_x = \frac{.015}{.06+.015} - (.016)(\frac{1}{.06+.015}) = -\frac{1}{75}$,

Since $Y = \frac{1-Z}{\delta} = \frac{1-Z}{.06}$, L can be written as $L = (1 + \frac{.016}{.06})Z - \frac{.016}{.06}$ and then
$Var[L] = (1 + \frac{.016}{.06})^2 Var[Z] = (1 + \frac{.016}{.06})^2({}^2\overline{A}_x - \overline{A}_x^2)$
$= (1 + \frac{.016}{.06})^2[\frac{.015}{.12+.015} - (\frac{.015}{.06+.015})^2] = .114094$.

If the total loss is denoted S, then $S = L_1 + L_2 + \cdots + L_n$ (sum of losses on each of the n policies), and $E[S] = nE[L] = -\frac{n}{75}$, and $Var[S] = nVar[L] = .114094n$.

Using the normal approximation, $P(S > 0) = P(\frac{S-E[S]}{\sqrt{Var[S]}} > \frac{-E[S]}{\sqrt{Var[S]}})$.
In order for this probability to be .05, we must have $\frac{-E[S]}{\sqrt{Var[S]}} = 1.645$,
or equivalently, $\frac{-(-\frac{n}{75})}{\sqrt{.114094n}} = 1.645$. Solving for n results in $n = 1737$.

The number of independent policies needed to have a positive loss probability of .025, say m, must satisfy the equation $\frac{-(-\frac{m}{75})}{\sqrt{.114094m}} = 1.96$, and solving for m results in $m = 2466$.

The additional number of policies needed is $2466 - 1737 = 729$.

Answer: D