

EXAM MLC QUESTIONS OF THE WEEK

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Week of February 5/07

Suppose that survival follows DeMoivre's law with $\omega = 100$, and $i = .08$.

Find \ddot{a}_{90} , ${}^2\ddot{a}_{90}$ and $Var[Y]$, where Y is the PVFV for a whole life annuity-due of 1 issued to (90).

The solution can be found below.

Week of February 5/07 - Solution

$$\begin{aligned}\ddot{a}_{90} &= \sum_{k=0}^{\infty} v^k {}_k p_{90} = \sum_{k=0}^9 v^k \left(\frac{10-k}{10} \right) = \left(\frac{1}{10} \right) (10 + 9v + 8v^2 + \dots + v^9) \\ &= \left(\frac{1}{10} \right) (D\ddot{a})_{\overline{10}|.08} = \left(\frac{1}{10} \right) \left(\frac{10 - a_{\overline{10}|.08}}{d} \right) = \left(\frac{1}{10} \right) \left(\frac{10 - 6.710}{.0741} \right) = 4.44 .\end{aligned}$$

$$\begin{aligned}{}^2\ddot{a}_{90} &= \sum_{k=0}^{\infty} v^{2k} {}_k p_{90} = \sum_{k=0}^9 v^{2k} \left(\frac{10-k}{10} \right) = \left(\frac{1}{10} \right) [10 + 9v^2 + 8(v^2)^2 + \dots + (v^2)^9] \\ &= \left(\frac{1}{10} \right) ({}^2D\ddot{a})_{\overline{10}|.08} = \left(\frac{1}{10} \right) \left(\frac{10 - {}^2a_{\overline{10}|}}{2d - d^2} \right) = \left(\frac{1}{10} \right) \left(\frac{10 - 4.720}{.1427} \right) = 3.70 .\end{aligned}$$

$$\text{Var}[Y] = \frac{2}{d} [\ddot{a}_{90} - {}^2\ddot{a}_{90}] + {}^2\ddot{a}_{90} - \ddot{a}_{90}^2 = 3.97 , \text{ or}$$

$$\text{Var}[Y] = \frac{1}{d^2} [{}^2A_{90} - A_{90}^2] = \left(\frac{1}{.0741} \right)^2 \left[\frac{{}^2a_{\overline{10}|}}{10} - \left(\frac{a_{\overline{10}|}}{10} \right)^2 \right] = 3.97 .$$