EXAM MLC QUESTIONS OF THE WEEK

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Week of April 9/07

(CAS) An auto insured who was claim-free during a policy period will be claim-free during the next policy period with probability 0.9. An auto insured who was not claim-free during a policy period will be claim-free during the next policy period with probability 0.7. What is the probability that an insured who was claim-free during the policy period 0 will incur a claim during policy period 3?
A) 0.120   B) 0 .124   C) 0.128   D) 0.132   E) 0.136

The solution can be found below.

Week of April 9/07 - Solution

We define state 0 to be claim-free and state 1 to be not claim-free. The one-step transition probability matrix is $P = \begin{bmatrix} .9 & .1 \\ .3 & .7 \end{bmatrix}$ (the values of .1 and .3 are the complements of the given transition probabilities, for instance, since there is a .9 chance that next period is claim-free given that this period is claim free, there must be a .1 chance that next period is not claim-free given that this period is claim-free). We wish to find $_3Q_0^{(0,1)} = P[X_3 = 1 | X_0 = 0]$.

The 3-step transition matrix is

$$Q \times Q \times Q = \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix} \times \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix} \times \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix} = \begin{bmatrix} .88 & .12 \\ .84 & .16 \end{bmatrix} \times \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix} = \begin{bmatrix} .876 & .124 \\ .868 & .132 \end{bmatrix}$$

$_3Q_0^{(0,1)} = .196$. Note that it was not necessary to do the entire matrix multiplication.

To get $_3Q_0^{(0,1)^3}$ in the final matrix multiplication we need only the top row of $Q \times Q$.

We could also try to find $_3Q_0^{(0,1)^3}$ by looking at all possible 3-step sequences from state 0 to state 1. These would be 0-0-0-1 , 0-0-1-1 , 0-1-0-1 , 0-1-1-1 , with total probability

$(.9)(.9)(.1) + (.9)(.1)(.3) + (.1)(.7)(.1) + (.1)(.3)(.3) = .124$.   Answer: B