EXAM FM QUESTIONS OF THE WEEK

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Week of May 8/06

A corporation has obligations to pay \$100,000 at the end of 2 years and another \$100,000 at the end of 3 years. There are 2 bonds available to buy to try to match these liabilities. The first bond is a 2 year bond with annual coupons at rate 20%, and the second bond is a 3 year bond with annual coupons at 40%. The yield rates on both bonds and reinvestment rates for each of the next 3 years are all at an annual effective rate of 25%. The corporation purchases α face amount of the 2 year bond and β face amount of the 3 year bond. The coupon received at time 1 is reinvested, and any excess of bond payment over 100,000 at time 2 is reinvested. The reinvested amount along with the bond payments at time 3 are exactly enough to pay the 100,000 due at time 3. Find the minimum value of α that guarantees that the asset cashflow available at time 2 is at least 100,000.

The solution can be found below.

Week of May 8/06 - Solution

Since the bond payments are just enough to pay the liabilities, it must be true that the combined present value of the bonds is equal to the present value of the liability payments. Therefore,
$\frac{.2\alpha+.4\beta}{1.25} + \frac{1.2\alpha+.4\beta}{(1.25)^2} + \frac{1.4\beta}{(1.25)^3} = 100,000[\frac{1}{(1.25)^2} + \frac{1}{(1.25)^3}]$.
We can write this as $.928\alpha + 1.2928\beta = 115,200$.

In order to meet the 100,000 liability payment at time 2, the reinvested bond payments at time 1 combined with the bond payments at time 2 must be at least 100,000 .
The bond payments at time 1 are $.2\alpha + .4\beta$. Reinvested for one year, they grow to $(.2\alpha + .4\beta)(1.25) = .25\alpha + .5\beta$. The additional bond payments at time 2 are $1.2\alpha + .4\beta$, so the total received at time 2 is $1.45\alpha + .9\beta$. This must be at least 100,000 .
From the equation $.928\alpha + 1.2928\beta = 115,200$ we have $\beta = \frac{115,200-.928\alpha}{1.2928}$,
from which we get $1.45\alpha + .9\beta = 80,198 + .8040\alpha \geq 100,000$.
This results in $\alpha \geq 24,629$.