EXAM C QUESTIONS OF THE WEEK

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Week of May 1/06

Semi-parametric empirical Bayesian credibility is being applied in the following situation.
The distribution of annual losses X on an insurance policy is uniform on the interval $(0, \theta)$, where θ has an unknown distribution. A sample of annual losses for 100 separate insurance policies is available. It is found that $\sum_{i=1}^{100} X_i = 200$ and $\sum_{i=1}^{100} X_i^2 = 600$.
For a particular insurance policy, it is found that the total losses over a 3 year period is 4.
Find the semi-parametric estimate of the losses in the 4th year for this policy.

Solution can be found below.

Week of May 1/06 - Solution

Hypothetical mean is $E(X|\theta) = \frac{\theta}{2}$.
Process variance is $Var(X|\theta) = \frac{\theta^2}{12}$.
Expected hypothetical mean is $\mu = E[X] = E[E(X|\theta)] = E(\frac{\theta}{2}) = \frac{1}{2}E[\theta]$,
Expected process variance $= v = E[Var(X|\theta)] = E[\frac{\theta^2}{12}] = \frac{1}{12}E[\theta^2]$.
Variance of hypothetical mean $= a = Var[E(X|\theta)] = Var(\frac{\theta}{2})$
$= \frac{1}{4}Var(\theta) = \frac{1}{4}[E(\theta^2) - (E(\theta))^2]$.

From the sample, we can estimate $E(X)$ as $\overline{X} = 2$, so this is also the estimate of $\frac{1}{2}E[\theta]$.
The estimate of $E[\theta]$ is 4.
From the sample we can estimate $Var(X)$ using the unbiased sample estimate,
$\frac{1}{99}[\Sigma X_i^2 - 100\overline{X}^2] = \frac{1}{99}[600 - 100(2^2)] = 2.02$.
But $Var(X) = a + v = \frac{1}{12}E[\theta^2] + \frac{1}{4}[E(\theta^2) - (E(\theta))^2] = \frac{1}{3}E[\theta^2] - \frac{1}{4}(E(\theta))^2$.
Using the estimated variance of X and the estimated mean of θ, we have
$2.02 = \frac{1}{3}E[\theta^2] - \frac{1}{4}(4^2)$, so that the estimate of $E[\theta^2]$ is 18.06 .
Then, $v = \frac{1}{12}E[\theta^2]$ is estimated to be 1.505 , and
$a = \frac{1}{4}[E(\theta^2) - (E(\theta))^2]$ is estimated to be .515 .

The estimate of losses in the 4th year is $\widehat{Z}\overline{Y} + (1 - \widehat{Z})\widehat{\mu}$
where $\widehat{Z} = \frac{3}{3+\frac{\widehat{v}}{\widehat{a}}} = \frac{3}{3+\frac{1.505}{.515}} = .507$, and $\widehat{\mu} = \overline{X} = 2$,
so that $\widehat{Z}\overline{Y} + (1 - \widehat{Z})\widehat{\mu} = (.507)(\frac{4}{3}) + (.493)(2) = 1.66$.