EXAM M QUESTIONS OF THE WEEK

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Week of May 15/06

Two independent non-homogeneous Poisson processes are being considered.
Process A has intensity function $\lambda_A(t) = \frac{1}{1+t}$ for $t \geq 0$
and Process B has intensity function $\lambda_B(t) = \frac{2}{1+t}$ for $t \geq 0$.
Suppose that one event of Type 1 has occurred by time t. Let the expected time at which that event occurred be denoted A_1. Now suppose that the two processes are combined into a single process with intensity function $\lambda_C(t) = \lambda_A(t) + \lambda_B(t)$, and suppose that one event in the combined process has occurred by time t. Let the expected time at which that event occurred be denoted C_1. Find $A_1 - C_1$.

The solution can be found below.

Week of May 15/06 - Solution

The conditional density of $S_1^{(1)}$ (time of first Type A event) given $N_1(t) = 1$ is $\frac{\lambda_1(s)}{m_1(t)} = \frac{1/(1+s)}{ln(1+t)}$ for $0 < s < t$. Then $A_1 = E[S_1^{(1)}|N_1(1) = 1] = \int_0^t s \cdot \frac{1/(1+s)}{ln(1+t)}\,ds = \frac{t}{ln(1+t)} - 1$.

The conditional density of S_1(time of first combined event) given $N(t) = 1$ is $\frac{\lambda(s)}{m(t)} = \frac{3/(1+s)}{3ln(1+t)} = \frac{1/(1+s)}{ln(1+t)}$ for $0 < s < t$.
Then $C_1 = E[S_1|N_1(1) = 1] = \int_0^t s \cdot \frac{1/(1+s)}{ln(1+t)}\,ds = \frac{t}{ln(1+t)} - 1$.
Therefore, $A_1 - C_1 = 0$.