

# EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2006

## Week of May 15/06

The following random sample of size 5 is taken from the distribution of  $X$ :

1 , 3 , 4 , 7 , 10

Bootstrap approximation of the mean square error of estimators is to be based on the following 6 resamplings of size 5 from the empirical distribution:

Resample 1 : 1 , 1 , 4 , 7 , 7

Resample 2 : 3 , 4 , 4 , 7 , 10

Resample 3 : 1 , 4 , 4 , 10 , 10

Resample 4 : 3 , 3 , 3 , 4 , 10

Resample 5 : 4 , 4 , 7 , 7 , 10

Resample 6 : 1 , 7 , 7 , 10 , 10

The median of  $X$  is estimated by the third order statistic of a sample.

Find the bootstrap approximation to the estimator of the median using the 6 resamplings.

**Solution can be found below.**

## Week of May 15/06 - Solution

The median of the empirical distribution is  $\theta = 4$ .

Resample	$\hat{\theta}_1$	$(\hat{\theta}_1 - 4)^2$
1, 1, 4, 7, 7	4	$(4 - 4)^2 = 0$
3, 4, 4, 7, 10	4	$(4 - 4)^2 = 0$
1, 4, 4, 10, 10	4	$(4 - 4)^2 = 0$
3, 3, 3, 4, 10	3	$(3 - 4)^2 = 1$
4, 4, 7, 7, 10	7	$(7 - 4)^2 = 9$
1, 7, 7, 10, 10	7	$(7 - 4)^2 = 9$

The bootstrap estimate of  $\text{MSE}(\hat{\theta})$  is  $\frac{0+0+0+1+9+9}{6} = 3.17$ .