EXAM P QUESTIONS OF THE WEEK

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A study of international athletes shows that of the two performance-enhancing steroids Dianabol and Winstrol, 5% of athletes use Dianabol and not Winstrol, 2% use Winstrol and not Dianabol, and 1% use both. A breath test has been developed to test for the presence of the these drugs in an athlete. Past use of the test has resulted in the following information regarding the accuracy of the test. Of the athletes that are using both drugs, the test indicates that 75% are using both drugs, 15% are using Dianabol only and 10% are using Winstrol only. In 80% of the athletes that are using Dianabol but not Winstrol, the test indicates they are using Dianabol but not Winstrol, and for the other 20% the test indicates they are using both drugs. In 60% of the athletes that are using Winstrol but not Dianabol, the test indicates that they are using Winstrol only, and for the other 40% the test indicates they are using both drugs. For all athletes that are using neither Dianabol nor Winstrol, the test always indicates that they are using neither drug. Of those athletes who test positive for Dianabol but not Winstrol, find the percentage that are using both drugs.

The solution can be found below.

Week of March/06 - Solution

We define the following events:
D - the athlete uses Dianabol
W - the athlete uses Winstrol
TD - the test indicates that the athlete uses Dianabol
TW - the test indicates that the athlete uses Winstrol

We are given the following probabilities
$P(D \cap W') = .05$, $P(D' \cap W) = .02$, $P(D \cap W) = .01$,
$P(TD \cap TW | D \cap W) = .75$, $P(TD \cap TW' | D \cap W) = .15$, $P(TD' \cap TW | D \cap W) = .1$,
$P(TD \cap TW | D \cap W') = .2$, $P(TD \cap TW' | D \cap W') = .8$,
$P(TD \cap TW | D' \cap W) = .4$, $P(TD' \cap TW | D' \cap W) = .6$.

We wish to find $P(D \cap W | TD \cap TW') = \frac{P(D \cap W \cap TD \cap TW')}{P(TD \cap TW')}$.

The numerator is $P(D \cap W \cap TD \cap TW') = P(TD \cap TW' | D \cap W) \cdot P(D \cap W)$
$= (.15)(.01) = .0015$.

The denominator is
$P(TD \cap TW') = P(TD \cap TW' \cap D \cap W) + P(TD \cap TW' \cap D' \cap W)$
$+ P(TD \cap TW' \cap D \cap W') + P(TD \cap TW' \cap D' \cap W')$
We have used the rule $P(A) = P(A \cap B_1) + P(A \cap B_2) + \cdots$, where $B_1, B_2, ...$
forms a partition. The partition in this case is $B_1 = D \cap W$, $B_2 = D' \cap W$,
$B_3 = D \cap W'$, $B_4 = D' \cap W'$, since an athlete must be using both, one or neither of the drugs.

We have just seen that $P(TD \cap TW' \cap D \cap W) = .0015$.
In a similar way, we have
$P(TD \cap TW' \cap D' \cap W) = P(TD \cap TW' | D' \cap W) \cdot P(D' \cap W) = (0)(.02) = 0$, and
$P(TD \cap TW' \cap D \cap W') = P(TD \cap TW' | D \cap W') \cdot P(D \cap W') = (.8)(.05) = .04$, and
$P(TD \cap TW' \cap D' \cap W') = P(TD \cap TW' | D' \cap W') \cdot P(D' \cap W') = (0)(.92) = 0$
(note that $P(D' \cap W') = 1 - P(D \cup W)$
$= 1 - P(D \cap W') - P(D' \cap W) - P(D \cap W) = .92$.

Then, $P(D \cap W | TD \cap TW') = \frac{.0015}{.0015+0+.04+0} = .036$, 3.6% .