EXAM FM QUESTIONS OF THE WEEK

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Week of March 27/06

Suppose that s_k denotes the annual effective yield on a zero coupon bond that matures in k years. For forward interest rates implied by the term structure of zero-coupon bond yields, suppose that $f_{n,n+1}$ denotes the one-year effective forward rate of interest in effect for the one year period starting at time n and ending at time $n+1$. Which of the following statements is (are) always true?

I. If $0 < s_1 < s_2 < \cdots < s_{k-1} < s_k$ then $f_{k-1,k} < s_k$

II. If the term structure is "flat" (i.e. constant s_k for all k), then the forward rate structure is flat.

The solution can be found below.

Week of March 27/06 - Solution

The relationship between zero-coupon yields and forward rates is

$$(1+s_n)^n(1+f_{n,n+1}) = (1+s_{n+1})^{n+1}$$

(n years of investment in a zero coupon bond followed by investment for 1 year at the forward rate is equivalent to $n+1$ years of investment in a zero coupon bond).

Then,

$(1+s_{k-1})^{k-1}(1+f_{k-1,k}) = (1+s_k)^k$

$\rightarrow 1+f_{k-1.k} = \frac{(1+s_k)^k}{(1+s_{k-1})^{k-1}} = \frac{(1+s_k)^{k-1}}{(1+s_{k-1})^{k-1}} \times (1+s_k) > 1+s_k$.

This is true because we have assumed $s_{k-1} < s_k$, and therefore $\frac{1+s_k}{1+s_{k-1}} > 1$.

It follows that $f_{k-1,k} > s_k$, so statement I is false.

If $s_k = s_{k+1} = s$, then $(1+s_{k+1})^{k+1} = (1+s_k)^k(1+f_{k,k+1})$

$\rightarrow 1+f_{k,k+1} = \frac{(1+s)^{k+1}}{(1+s)^k} = 1+s \rightarrow f_{k,k+1} = s$ for all k.

Statement II is true.