EXAM P QUESTIONS OF THE WEEK

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An insurance company does a study of claims that arrive at a regional office. The study focuses on the days during which there were at most 2 claims. The study finds that for the days on which there were at most 2 claims, the average number of claims per day is 1.2 . The company models the number of claims per day arriving at that office as a Poisson random variable. Based on this model, find the probability that at most 2 claims arrive at that office on a particular day.

The solution can be found below.

Week of March 20/06 - Solution

Suppose that the mean number of claims per day arriving at the office is λ.
Let X denote the number of claims arriving in one day.
Then the probability of at most 2 claims in one day is $P(X \leq 2) = e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2}$.
The conditional probability of 0 claims arriving on a day given that there are at most 2 for the day is $P(X = 0|X \leq 2) = \frac{P(X=0)}{P(X\leq 2)} = \frac{e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}+\frac{\lambda^2 e^{-\lambda}}{2}} = \frac{1}{1+\lambda+\frac{\lambda^2}{2}}$.
The conditional probability of 1 claim arriving on a day given that there are at most 2 for the day is $P(X = 1|X \leq 2) = \frac{P(X=1)}{P(X\leq 2)} = \frac{\lambda e^{-\lambda}}{e^{-\lambda}+\lambda e^{-\lambda}+\frac{\lambda^2 e^{-\lambda}}{2}} = \frac{\lambda}{1+\lambda+\frac{\lambda^2}{2}}$.
The conditional probability of 2 claims arriving on a day given that there are at most 2 for the day is $P(X = 2|X \leq 2) = \frac{P(X=2)}{P(X\leq 2)} = \frac{\frac{\lambda^2 e^{-\lambda}}{2}}{e^{-\lambda}+\lambda e^{-\lambda}+\frac{\lambda^2 e^{-\lambda}}{2}} = \frac{\frac{\lambda^2}{2}}{1+\lambda+\frac{\lambda^2}{2}}$.
The expected number of claims per day, given that there were at most 2 claims per day is
$(0)(\frac{1}{1+\lambda+\frac{\lambda^2}{2}}) + (1)(\frac{\lambda}{1+\lambda+\frac{\lambda^2}{2}}) + (2)(\frac{\frac{\lambda^2}{2}}{1+\lambda+\frac{\lambda^2}{2}}) = \frac{\lambda+\lambda^2}{1+\lambda+\frac{\lambda^2}{2}}$.
We are told that this is 1.2 .
Therefore $\lambda + \lambda^2 = (1.2)(1 + \lambda + \frac{\lambda^2}{2})$, which becomes the quadratic equation $.4\lambda^2 - .2\lambda - 1.2 = 0$. Solving the equation results in $\lambda = 2$ or -1.5 , but we ignore the negative root. The probability of at most 2 claims arriving at the office on a particular day is $P(X \leq 2) = e^{-2} + 2e^{-2} + \frac{2^2 e^{-2}}{2} = .6767$.