EXAM C QUESTIONS OF THE WEEK

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Week of March 20/06

A portfolio of insurance policies consists of two types of policies. The annual aggregate loss distribution for each type of policy is a compound Poisson distribution. Policies of Type I have a Poisson parameter of 1 and policies of Type 2 have a Poisson parameter of 2. For both policy types, the claim size (severity) distribution is uniformly distributed on the integers 1, 2 and 3. Half of the policies are of Type I and half are of Type II.

A policy is chosen at random and an aggregate annual claim of 2 is observed.
Find the posterior distribution of the Poisson parameter.

Solution can be found below.

Week of March 20/06 - Solution

The prior parameter λ has distribution $\lambda = \begin{cases} 1 & \text{prob. } \frac{1}{2} \\ 2 & \text{prob. } \frac{1}{2} \end{cases}$.

The model distribution S has a compound distribution with Poisson frequency with mean λ, and the stated severity distribution.

(a) $P(\lambda = 1|S_1 = 2) = \frac{P(S_1=2\cap\lambda=1)}{P(S_1=2)}$

$P(S_1 = 2|\lambda = 1) = P(1 \text{ claim for amount } 2|\lambda = 1) + P(2 \text{ claims for amount 1 each}|\lambda = 1)$
$= e^{-1} \cdot \frac{1}{3} + \frac{e^{-1}}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{7e^{-1}}{18}$.

$P(S_1 = 2 \cap \lambda = 1) = P(S_1 = 2|\lambda = 1) \cdot P(\lambda = 1) = \frac{7e^{-1}}{18} \cdot \frac{1}{2} = \frac{7e^{-1}}{36}$.

$P(S_1 = 2|\lambda = 2) = P(1 \text{ claim for amount } 2|\lambda = 2) + P(2 \text{ claims for amount 1 each}|\lambda = 2)$
$= e^{-2} \cdot 2 \cdot \frac{1}{3} + \frac{e^{-2}\cdot 2^2}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{8e^{-2}}{9}$.

$P(S_1 = 2 \cap \lambda = 2) = P(S_1 = 2|\lambda = 2) \cdot P(\lambda = 2) = \frac{8e^{-2}}{9} \cdot \frac{1}{2} = \frac{4e^{-2}}{9}$.

$P(S_1 = 2) = P(S_1 = 2 \cap \lambda = 1) + P(S_1 = 2 \cap \lambda = 2) = \frac{7e^{-1}}{36} + \frac{4e^{-2}}{9}$.

$P(\lambda = 1|S_1 = 2) = \frac{P(S_1=2\cap\lambda=1)}{P(S_1=2)} = (\frac{7e^{-1}}{36}) \Big/ (\frac{7e^{-1}}{36} + \frac{4e^{-2}}{9}) = .5432$
and $P(\lambda = 2|S_1 = 2) = 1 - P(\lambda = 1|S_1 = 2) = .4568$.