EXAM M QUESTIONS OF THE WEEK

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Week of March 13/06

X has an exponential distribution with mean 1.
Y is a transformation of X based on the increasing one-to-one transformation $Y = g(X)$.
The distribution of Y is Weibull with parameters τ and θ.
Find the transformation function g.

The solution can be found below.

Week of March 13/06 - Solution

Since the transformation is one-to-one and increasing, we have
$F_Y(y) = 1 - e^{-(y/\theta)^\tau} = P(Y \leq y) = P(g(X) \leq y) = P(X \leq g^{-1}(y)) = 1 - e^{-g^{-1}(y)}$,
where $g^{-1}(y)$ is the inverse function of g.

It follows that $x = g^{-1}(y) = (\frac{y}{\theta})^\tau$, from which we get $y = \theta x^{1/\tau} = g(x)$.

Alternatively, let us define $k(y) = g^{-1}(y)$.
The mechanical transformation approach gives us $f_Y(y) = f_X(k(y)) \cdot |k'(y)|$.
Therefore, $\frac{\tau\theta^\tau}{y^{\tau+1}} e^{-(y/\theta)^\tau} = e^{-k(y)} \cdot |k'(y)|$.
It appears from this relationship that $x = k(y) = (\frac{y}{\theta})^\tau$, and the $y = \theta x^{1/\tau} = g(x)$.
Trying this transformation results in the correct distribution for Y.