

EXAM C QUESTIONS OF THE WEEK

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Week of March 13/06

N is the distribution of the number of claims occurring per week. N has a Poisson distribution with an unknown mean. The standard for full credibility for N is based on the sample mean of N being within 5% of the true mean of N with probability 90%.

With 400 observed claims in 20 weeks, the credibility premium based on partial credibility is P . With 500 observed claims in 30 weeks, the credibility premium based on partial credibility is $P - 1.91$. Find the credibility premium based on partial credibility if there are 550 observed claims in 35 weeks. Assume that the same manual premium is used in all cases.

Solution can be found below.

Week of March 13/06 - Solution

Since N is Poisson, the full credibility standard for estimating the mean of N is either

(i) $1082.4 \cdot \frac{Var(N)}{[E(N)]^2} = 1082.4 \cdot \frac{\lambda}{\lambda^2} = \frac{1082.4}{\lambda}$ as the expected number of exposures of N (weeks) needed,

or

(ii) $1082.4 \cdot \frac{Var(N)}{E(N)} = 1082.4 \cdot \frac{\lambda}{\lambda} = 1082.4$ as the total expected number of claims needed.

Since we do not know the value of λ , the only standard we can apply is (ii).

With 400 claims in 20 weeks, the average number of claims per week (sample mean) is $\bar{N} = \frac{400}{20} = 20$

. Using credibility standard (ii) above, the partial credibility factor is

$$Z = \sqrt{\frac{400}{1082.4}} = .6079, \text{ and the partial credibility premium is}$$

$$Z \cdot \bar{N} + (1 - Z) \cdot M = 12.16 + .3921M = P, \text{ where } M \text{ is the manual premium.}$$

With 500 claims in 30 weeks, the average number of claims per week (sample mean) is

$\bar{N} = \frac{500}{30} = 16.6667$. Using credibility standard (ii) above, the partial credibility factor is

$$Z = \sqrt{\frac{500}{1082.4}} = .6797, \text{ and the partial credibility premium is}$$

$$Z \cdot \bar{N} + (1 - Z) \cdot M = 11.33 + .3203M = P - 1.91, \text{ where } M \text{ is the manual premium.}$$

From the two equations, $12.16 + .3921M = P$ and $11.33 + .3203M = P - 1.91$,

we get $M = 15.04$ and $P = 18.04$. Then, with 550 claims in 35 weeks, we have

$\bar{N} = \frac{550}{35} = 15.7143$. Using credibility standard (ii) above, the partial credibility factor is

$$Z = \sqrt{\frac{550}{1082.4}} = .7128, \text{ and the partial credibility premium is}$$

$$Z \cdot \bar{N} + (1 - Z) \cdot M = 11.20 + (.2872)(15.04) = 15.52.$$