

EXAM P QUESTIONS OF THE WEEK

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Week of June 19/06

In the 2006 World Cup of soccer, according to an online ranking service, Brazil, England and Germany are the three most highly ranked teams to win the tournament. A survey of soccer fans asks the fans to rank from most likely to least likely the chance of each those country's teams winning the world cup. The survey found the following:

- $\frac{2}{3}$ of those who ranked Germany first ranked Brazil second ,
- $\frac{1}{7}$ of those who didn't rank Germany first ranked Brazil second ,
- 30% of those surveyed ranked Brazil second.

Of those surveyed who ranked Brazil second, find the proportion that ranked Germany third.

The solution can be found below.

Week of June 19/06 - Solution

We define the following events:

$B2$ - a surveyed individual ranked Brazil second ,

$G1$ - a surveyed individual ranked Germany first .

We wish to find $P(G3|B2) = \frac{P(B2 \cap G3)}{P(B2)}$.

We are given $P(B2) = .3$,

and we are given the conditional probabilities $P(B2|G1) = \frac{2}{3}$ and $P(B2|G1') = \frac{1}{7}$.

From $P(B2|G1) = \frac{P(B2 \cap G1)}{P(G1)} = \frac{2}{3}$ we get $P(B2 \cap G1) = \frac{2}{3} \cdot P(G1)$, and from

$P(B2|G1') = \frac{1}{7}$ we get $P(B2 \cap G1') = \frac{1}{7} \cdot P(G1') = \frac{1}{7} \cdot [1 - P(G1)]$.

Therefore $.3 = P(B2) = P(B2 \cap G1) + P(B2 \cap G1') = \frac{2}{3} \cdot P(G1) + \frac{1}{7} \cdot P(G1')$
 $= \frac{2}{3} \cdot P(G1) + \frac{1}{7} \cdot [1 - P(G1)]$, from which we get $P(G1) = .3$.

Then, $P(B2 \cap G3) = P(B2 \cap G1') = \frac{1}{7} \cdot [1 - P(G1)] = .1$

and $P(G3|B2) = \frac{P(B2 \cap G3)}{P(B2)} = \frac{.1}{.3} = \frac{1}{3}$.