

EXAM C QUESTIONS OF THE WEEK

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Week of June 19/06

You are given the following random sample of 6 observations from the distribution of the random variable X :

2 , 4 , 4 , 5 , 7 , 10

Kernel smoothing is applied to estimate the density function of X . The kernel function used for the data point y is the pdf of the normal distribution with mean y and variance 1. Use kernel smoothing to estimate the distribution function of X at the point $x = 3$, $\hat{F}(3)$.

Solution can be found below.

Week of June 19/06 - Solution

The data points are $y_1 = 2$, $y_2 = 4$, $y_3 = 5$, $y_4 = 7$, $y_5 = 10$.

The empirical distribution is $p(2) = \frac{1}{6}$, $p(4) = \frac{1}{3}$, $p(5) = \frac{1}{6}$, $p(7) = \frac{1}{6}$, $p(10) = \frac{1}{6}$

The kernel smoothed estimate of $F(3)$ is

$$\begin{aligned}\widehat{F}(3) &= \sum_{j=1}^k p(y_j) \cdot K_{y_j}(3) \\ &= \frac{1}{6} \cdot K_2(3) + \frac{1}{3} \cdot K_4(3) + \frac{1}{6} \cdot K_5(3) + \frac{1}{6} \cdot K_7(3) + \frac{1}{6} \cdot K_{10}(3),\end{aligned}$$

where $K_y(x)$ is the cdf of the normal distribution with mean y and variance 1.

$$K_y(x) = \Phi\left(\frac{x-y}{1}\right) = \Phi(x-y).$$

$$K_2(3) = \Phi(3-2) = \Phi(1) = .8413,$$

$$K_4(3) = \Phi(3-4) = \Phi(-1) = .1587,$$

$$K_5(3) = \Phi(3-5) = \Phi(-2) = .0228,$$

$$K_7(3) = \Phi(3-7) = \Phi(-4) = 0,$$

$$K_{10}(3) = \Phi(3-10) = \Phi(-7) = 0.$$

$$\begin{aligned}\text{Then, } \widehat{F}(3) &= \frac{1}{6} \cdot K_2(3) + \frac{1}{3} \cdot K_4(3) + \frac{1}{6} \cdot K_5(3) + \frac{1}{6} \cdot K_7(3) + \frac{1}{6} \cdot K_{10}(3) \\ &= \frac{1}{6} \cdot (.8413) + \frac{1}{3} \cdot (.1587) + \frac{1}{6} \cdot (.0228) + \frac{1}{6} \cdot (0) + \frac{1}{6} \cdot (0) = .1969.\end{aligned}$$