EXAM M QUESTIONS OF THE WEEK

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Week of July 3/06

An insurer is calculating the single premium to charge for continuous whole life insurance to (80) with face amount 100,000. The insurer calculates the premium in the following way.

Using $\delta = .05$ and DeMoivre's Law with $\omega = 100$ a continuous single premium whole life insurance policy with face amount 100,000 is issued to each of 100 individuals at age 81, and the single premium charged results in a probability of .1587 (based on the normal approximation) of the insurer experiencing a positive loss on all policies combined.

That is the premium charged to (80) by the insurer. Using the same interest and mortality assumptions, suppose that this insurance policy is issued to 100 independent individuals at age (80). Using the normal approximation, find the probability of a positive loss on all policies combined.

The solution can be found below.

Week of July 3/06 - Solution

Under DeMoivre's Law, $\overline{A}_x = \frac{1}{\omega-x} \cdot \overline{a}_{\overline{\omega-x}|} = \frac{1-e^{-(\omega-x)\delta}}{(\omega-x)\delta}$,
and ${}^2\overline{A}_x = \frac{1}{\omega-x} \cdot {}^2\overline{a}_{\overline{\omega-x}|} = \frac{1-e^{-(\omega-x)2\delta}}{(\omega-x)2\delta}$.
With $\delta = .05$, $\omega = 100$ we have $\overline{A}_{80} = \frac{\overline{a}_{\overline{20}|}}{20} = .632121$, $\overline{A}_{81} = \frac{\overline{a}_{\overline{19}|}}{19} = .645536$
${}^2\overline{A}_{80} = \frac{{}^2\overline{a}_{\overline{20}|}}{20} = 432332$ and ${}^2\overline{A}_{81} = \frac{{}^2\overline{a}_{\overline{19}|}}{19} = 447595$.

The loss on a single policy with premium Q is $L = 100,000Z - Q$, where $Z = v^T$ is the present value random variable of a continuous insurance of 100,000 issued to (81).
We have $E[L] = 100,000\overline{A}_{81} - Q = 64,553.6 - Q$, and
$Var[L] = 100,000^2 \cdot Var[Z] = 100,000^2 \cdot ({}^2\overline{A}_{81} - \overline{A}_{81}^2) = 100,000^2(.030878)$.

The combined loss on 100 policies is $S = \sum_{j=1}^{100} L_j$, where each L_j has the same distribution as L but the L_j's are independent of one another. We wish to find Q so that $P[S > 0] = .10$.

We have $E[S] = 100\, E[L] = 100(64,553.6 - Q)$ and it follows from independence of the L_j's that $Var[S] = 100\, Var[L] = 10^2 \cdot 100,000^2(.030878)$.

$P[S > 0] = P[\frac{S-100(64,553.6-Q)}{\sqrt{10^2 \cdot 100,000^2(.030878)}} > \frac{100(Q-64,553.6)}{\sqrt{10^2 \cdot 100,000^2(.030878)}}] = .1587 = 1 - \Phi(1)$.
Applying the normal approximation to S, it follows that $\frac{100(Q-64,553.6)}{\sqrt{10^2 \cdot 100,000^2(.030878)}} = 1$,
so that $Q = 66,310.8$. This is the premium charged to each of 100 individuals at age (80).

Now the loss on a single policy with premium Q is $L = 100,000Z - Q$, where $Z = v^T$ is the present value random variable of a continuous insurance of 100,000 issued to (80).
We have $E[L] = 100,000\overline{A}_{80} - Q = 63,212.1 - 66,310.8 = -3,098.7$, and
$Var[L] = 100,000^2 \cdot Var[Z] = 100,000^2 \cdot ({}^2\overline{A}_{80} - \overline{A}_{80}^2) = 100,000^2(.032755) =$.

The combined loss on 100 policies is $S = \sum_{j=1}^{100} L_j$, with
$E[S] = 100\, E[L] = -309,870$ and $Var[S] = 100\, Var[L] = 10^2 \cdot 100,000^2(.032755)$.

Then $P[S > 0] = P[\frac{S-(-309,870)}{\sqrt{10^2 \cdot 100,000^2(.032755)}} > \frac{309,870}{\sqrt{10^2 \cdot 100,000^2(.032755)}}] = 1 - \Phi(1.71) = .0436$.