EXAM M QUESTIONS OF THE WEEK

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Week of July 31/06

A 4-payment, 5-year fully discrete term insurance issued to (x) has a death benefit of $100,000 if death occurs within the first 3 years, and $50,000 if death occurs in the 4th year or 5th year. The annual benefit premium payable for 4 years is $2,379.30 .

Formulate the 2nd year terminal prospective loss function as a 4-point random variable.

Given that $i = .08$ and $_{k|}q_x = .025$ for $k = 0, 1, 2, ...$ find $_2V$, the 2nd year terminal benefit reserve for this policy.

The solution can be found below.

Week of July 31/06 - Solution

At the end of the 2nd year there are 3 years remaining on the policy.
One of 4 events must occur to $(x+2)$:
(i) death occurs before age $x+3$,
(ii) death occurs between $x+3$ and $x+4$,
(iii) death occurs between $x+4$ and $x+5$,
(iv) survival to age $x+5$.
${}_2L$ can be represented as the 4-point random variable

$$ {}_2L = \begin{cases} 100,000v - 2379.30 = 90,213.29 & K(x+2)=0 \quad \text{prob. } q_{x+2} \\ 50,000v^2 - 2379.30(1+v) = 38,284.59 & K(x+2)=1 \quad \text{prob. } {}_{1|}q_{x+2} \\ 50,000v^3 - 2379.30(1+v) = 35,109.26 & K(x+2)=2 \quad \text{prob. } {}_{2|}q_{x+2} \\ 0 - 2379.30(1+v) = -4,582.36 & K(x+2)\geq 3 \quad \text{prob. } {}_3p_{x+2} \end{cases} $$

Since ${}_{k|}q_x = .025$ for $k=0,1,2,\ldots$, it follows that ${}_2q_x = q_x + {}_{1|}q_x = .05 = \frac{1}{20}$, and ${}_2p_x = \frac{19}{20}$. Then, since ${}_{k+2|}q_x = {}_2p_x \cdot {}_{k|}q_{x+2}$ for $k=0,1,2,\ldots$, it follows that ${}_{k|}q_{x+2} = {}_{k+2|}q_x / {}_2p_x = (.025)/(\frac{19}{20}) = \frac{1}{38}$ for $k=0,1,2,\ldots$.

Then, $q_{x+2} = \frac{1}{38}$, ${}_{1|}q_{x+2} = \frac{1}{38}$, ${}_{2|}q_{x+2} = \frac{1}{38}$, and ${}_3q_{x+2} = \; = q_{x+2} + {}_{1|}q_{x+2} + {}_{2|}q_{x+2} = \frac{3}{38}$ and ${}_3p_{x+2} = \frac{35}{38}$.

${}_2V = E[{}_2L|(x)$ alive at age $x+2]$
$= (90,213.29)(\frac{1}{38}) + (38,284.59)(\frac{1}{38}) + (35,109.26)(\frac{1}{38}) + (-4,582.36)(\frac{35}{38}) = 84.86$.