EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2005

Question 1 - Week of July 25

A select-and-ultimate life table has a select period of 10 years. The force of mortality is $\mu_{[x]}(t) = .01$ during the select period, and the force of mortality is $\mu(y) = .02$ for some at age y who is past the select period. Find an expression for the complete expectation $\mathring{e}_{[x]+t}$ for $t \geq 0$.

The solution can be found below.

Question 1 - Week of July 25 - Solution

We first note that $\overset{\circ}{e}_y = \frac{1}{.02} = 50$ for $y \geq [x] + 10$.

$\overset{\circ}{e}_{[x]+t} = \int_0^\infty {}_sp_{[x]+t}\, ds$ for $0 \leq t < 10$ and $\overset{\circ}{e}_{[x]+t} = \overset{\circ}{e}_{x+t}$ for $t \geq 10$.
${}_sp_{[x]+t} = exp[-\int_0^s \mu_{[x]}(t+r)\, dr] = e^{-.01s}$ for $s < 10 - t$.

For $t < 10$, we can write $\overset{\circ}{e}_{[x]+t} = \overset{\circ}{e}_{[x]+t:\overline{10-t}|} + {}_{10-t}p_{[x]+t} \cdot e_{[x]+10}$.
It was noted above that $e_{[x]+10} = 50$, and ${}_{10-t}p_{[x]+t} = e^{-.01(10-t)}$.
And $\overset{\circ}{e}_{[x]+t:\overline{10-t}|} = \int_0^{10-t} {}_sp_{[x]+t}\, ds = \int_0^{10-t} e^{-.01s}\, ds = \frac{1-e^{-.01(10-t)}}{.01}$.
Then for $t < 10$, $\overset{\circ}{e}_{[x]+t} = \frac{1-e^{-.01(10-t)}}{.01} + e^{-.01(10-t)} \cdot 50 = 100 - 50e^{-.01(10-t)}$.