EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of July 10/06

Z_1 is the present value random variable for a continuous n-year certain and life annuity of 1 per year issued to (x).

Z_2 is the present value random variable for a continuous whole life annuity of 1 per year issued to (x).

The force of mortality is constant at $\mu(y) = \mu > 0$ for all y, and the force of interest is $\delta > 0$.

Find each of the following in terms of μ and δ or annuity notation:

(a) $P(Z_1 > Z_2)$

(b) $\int_0^\infty [F_1(t) - F_2(t)]\, dt$, where F_1 and F_2 are the distribution functions of Z_1 and Z_2, respectively.

The solution can be found below.

Week of July 10/06 - Solution

(a) If (x) lives beyond n years, then $Z_1 = Z_2$, since both annuities will pay up to the time of (x)'s death. If (x) dies at time $t < n$, then $Z_1 = \overline{a}_{\overline{n}|}$ and $Z_2 = \overline{a}_{\overline{t}|}$, so that $Z_2 > Z_2$. Therefore, $P(Z_1 > Z_2) = P[(x)$ dies before time $n] = 1 - {}_np_x = 1 - e^{-\mu n}$.

(b) Since Z_1 and Z_2 are non-negative random variables with finite means, it is true that $E[Z_1] = \int_0^\infty [1 - F_1(t)]\,dt$ and $E[Z_2] = \int_0^\infty [1 - F_2(t)]\,dt$.

Then,

$\int_0^\infty [F_1(t) - F_2(t)]\,dt \int_0^\infty [(1 - F_2(t)) - (1 - F_1(t))]\,dt$

$= E[Z_2] - E[Z_1] = \overline{a}_x - \overline{a}_{\overline{x:\overline{n}|}} = \overline{a}_x - (\overline{a}_x + \overline{a}_{\overline{n}|} - \overline{a}_{x:\overline{n}|}) = -\overline{a}_{\overline{n}|} + \overline{a}_{x:\overline{n}|}$.