EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2006

Week of July 10/06

A mortality study takes place over two years. Individuals are by one year integer age intervals. The following table summarizes the data collected. An interval of the form $(x, x+1]$ indicates that the individuals in that interval are classified as being age x at last birthday.

	Age When the Study Begins			
Age at death	$(0,1]$	$(1,2]$	$(2,3]$	Total
$(0,1]$	15	–	–	15
$(1,2]$	20	15	–	35
$(2,3]$	10	10	5	25
$(3,4]$	–	10	5	15
$(4,5]$	–	–	5	5
Age of Censoring				
$(0,1]$	10	–	–	10
$(1,2]$	30	20	–	50
$(2,3]$	915	10	5	930
$(3,4]$	–	1135	5	1140
$(4,5]$	–	–	775	775
Total	1000	1200	800	3000

The Kaplan-Meier approximation for large data sets is used to estimate $S(2)$.
An actuary formulates an estimate for $S(2)$ as the function $f(\alpha, \beta)$ of the parameters α and β that are used in the estimation procedure.

(a) Find $f(1,0)$.

(b) Find $\frac{\partial}{\partial \beta} f(\alpha, \beta) \Big|_{\alpha=1, \beta=0}$.

Solution can be found below.

Week of July 10/06 - Solution

Using the notation for the Kaplan-Meier approximation for large data sets, we have intervals based on $c_0 = 0$, $c_1 = 1$, $c_2 = 2$, . . . , and
$d_0 = 1000$, $d_1 = 1200$, $d_2 = 800$, $x_0 = 15$, $x_1 = 35$, $x_2 = 25$, $x_3 = 15$, $x_4 = 5$,
$u_0 = 10$, $u_1 = 50$, $u_2 = 930$, $u_3 = 1140$, $u_4 = 775$.

With parameters α and β, we have $r_0 = \alpha d_0 - \beta u_0 = 1000\alpha - 10\beta$,
and $r_1 = (d_0 - u_0 - x_0) + \alpha d_1 - \beta u_1 = 975 + 1200\alpha - 50\beta$.

$S(2) = f(\alpha, \beta) = (1 - \frac{x_0}{r_0})(1 - \frac{x_1}{r_1}) = (1 - \frac{15}{1000\alpha - 10\beta})(1 - \frac{35}{975 + 1200\alpha - 50\beta})$

(a) Then $f(1, 0) = (1 - \frac{15}{1000})(1 - \frac{35}{2175}) = .96915$.

(b) $\frac{\partial}{\partial\beta} f(\alpha, \beta) = -\frac{150}{(1000\alpha - 10\beta)^2} \cdot (1 - \frac{35}{975 + 1200\alpha - 50\beta}) + (1 - \frac{15}{1000\alpha - 10\beta}) \cdot [-\frac{1750}{(975 + 1200\alpha - 50\beta)^2}]$
Substituting $\alpha = 1$, $\beta = 0$ results in a value of $-.000512$.