# EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2006

## Week of January 9/06

Two 60-year old individuals have independent future lifetimes, but both have survival based on DeMoivre's Law with $\omega = 100$. Annual effective interest is 6%. Find the actuarial present value of a continuous whole life annuity that pays at a rate of 3 per year until the first death, and after the first death continues at a rate of 1 per year until the second death.

**The solution can be found below.**

## Week of January 9/06 - Solution

We wish to find $2\overline{a}_{50:50} + \overline{a}_{\overline{50:50}} = 2\overline{a}_{50:50} + \overline{a}_{50} + \overline{a}_{50} - \overline{a}_{50:50} = \overline{a}_{50:50} + 2\overline{a}_{50}$ .

For DeMoivre's Law with upper age limit $\omega$, we have
$\overline{A}_x = \int_0^{\omega-x} v^t \cdot {}_tp_x \cdot \mu_x(t)\,dt = \int_0^{\omega-x} v^t \cdot \frac{\omega-x-t}{\omega-x} \cdot \frac{1}{\omega-x-t}\,dt = \int_0^{\omega-x} v^t \cdot \frac{1}{\omega-x}\,dt = \frac{\overline{a}_{\overline{\omega-x}|}}{\omega-x}$ .
Then, $\overline{a}_x = \frac{1-\overline{A}_x}{\delta}$ .
Therefore, $\overline{A}_{60} = \frac{\overline{a}_{\overline{100-60}|}}{100-60} = \frac{\overline{a}_{\overline{40}|}}{40} = .387333$ , and $\overline{a}_{60} = 10.5145$ .

$\overline{a}_{xx} = \frac{1-\overline{A}_{xx}}{\delta}$ , and under DeMoivre's Law, with independent lives both of age $x$,
$\overline{A}_{xx} = \int_0^{\omega-x} v^t \cdot {}_tp_{xx} \cdot \mu_{xx}(t)\,dt = \int_0^{\omega-x} v^t \cdot (\frac{\omega-x-t}{\omega-x})^2 \cdot \frac{2}{\omega-x-t}\,dt$ ,
since for independent lives, $\mu_{xx}(t) = \mu_x(t) + \mu_x(t)$ .
This integral can be written as $\frac{2}{\omega-x} \cdot \int_0^{\omega-x} v^t \cdot (\frac{\omega-x-t}{\omega-x})\,dt$ ,
which is $\frac{2}{\omega-x} \cdot \overline{a}_x$ .
Therefore, in this case, $\overline{A}_{60:60} = \frac{2}{100-60} \cdot \overline{a}_{60} = \frac{2}{40} \cdot (10.5145) = .525725$ ,
and $\overline{a}_{50:50} = \frac{1-\overline{A}_{50:50}}{\delta} = 8.1394$ .

The APV of the annuity is $8.1394 + 2(10.5145) = 37.31$ .