EXAM P QUESTIONS OF THE WEEK

S. Broverman, 2005

Week of January 30/06

X and Y are independent continuous random variables. They have the same distribution functions, $F_X(t) = 1 - \frac{1}{t}$ for $t > 1$ and $F_Y(t) = 1 - \frac{1}{t}$ for $t > 1$.
We define two new random variables W and Z as follows:
$W = min\{X, Y\}$ and $Z = max\{X, Y\}$.
Find the mean of W and the mean of Z.

The solution can be found below.

Week of January 30/06 - Solution

The pdf of X is $f_X(t) = F'_X(t) = \frac{1}{t^2}$, and the pdf of Y is the same, $f_Y(t) = \frac{1}{t^2}$.

The pdf of W is $f_W(t) = F'_W(t)$. We can find the pdf of W as follows.
$F_W(t) = P[W \leq t] = P[min\{X.Y\} \leq t] = 1 - P[min\{X, Y\} > t]$
$= 1 - P[(X > t) \cap (Y > t)] = 1 - P[X > t] \cdot P[Y > t]$
$= 1 - (\frac{1}{t})(\frac{1}{t}) = 1 - \frac{1}{t^2}$.
Then, $f_W(t) = \frac{2}{t^3}$ for $t > 1$, and $E[W] = \int_1^\infty t \cdot \frac{2}{t^3}\, dt = 2$.

The pdf of Z is $f_Z(t) = F'_Z(t)$. We can find the pdf of Z as follows.
$F_Z(t) = P[Z \leq t] = P[max\{X.Y\} \leq t] = P[max\{X, Y\} \leq t]$
$= P[(X \leq t) \cap (Y \leq t)] = P[X \leq t] \cdot P[Y \leq t]$
$= (1 - \frac{1}{t})(1 - \frac{1}{t}) = 1 - \frac{2}{t} + \frac{1}{t^2}$.
Then, $f_Z(t) = \frac{2}{t^2} - \frac{2}{t^3}$ for $t > 1$, and $E[Z] = \int_1^\infty t \cdot (\frac{2}{t^2} - \frac{2}{t^3})\, dt = \infty$
(since $\int_1^\infty t \cdot \frac{2}{t^2}\, dt = \infty$ and $\int_1^\infty t \cdot \frac{2}{t^3}\, dt = 2$).