

EXAM M QUESTIONS OF THE WEEK

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Week of January 2/06

The following information is given for a fully discrete whole life insurance issued to (x) with face amount b and level benefit premium π payable for life (k is an integer):

$$i = .08, \quad {}_kV = 63.1724, \quad {}_{k+1}V = 70.1614, \quad \pi = 11.0396,$$

$$\text{Var}[{}_kL|K(x) \geq k] = 3671.11, \quad \text{Var}[{}_{k+1}L|K(x) \geq k+1] = 3342.04.$$

Find the annual benefit premium for this policy.

The solution can be found below.

Week of January 2/06 - Solution

This is an exercise in combining the recursive relationship for benefit reserves with the recursive relationship for the variance of the prospective loss random variable.

The recursive relationship for benefit reserves is

$$({}_kV + \pi_k)(1 + i) - (b_{k+1} - {}_{k+1}V)q_{x+k} = {}_{k+1}V,$$

and the recursive relationship for loss variable variance is

$$\text{Var}[{}_kL|K(x) \geq k] = [v(b_{k+1} - {}_{k+1}V)]^2 p_{x+k} q_{x+k} + v^2 p_{x+k} \text{Var}[{}_{k+1}L|K(x) \geq k + 1].$$

From the first equation, we get

$$(63.1724 + 11.0396)(1.08) - (b - 70.1614)q_{x+k} = 70.1614,$$

$$\text{so that } (b - 70.1614)q_{x+k} = 9.98756.$$

Substituting this into the second equation, we get

$$3671.11 = [v(\frac{9.98756}{q_{x+k}})]^2(1 - q_{x+k})q_{x+k} + v^2(1 - q_{x+k})(3342.04), \text{ which becomes}$$

$$3671.11 = \frac{85.5207}{q_{x+k}}(1 - q_{x+k}) + 2865.26(1 - q_{x+k}).$$

After multiplying through by q_{x+k} we get the quadratic equation

$$2865.26q^2 + 891.37q - 85.52 = 0.$$

The equation has two roots, one negative, which we ignore. The positive root is $q = .0769$.

From the relationship $(b - 70.1614)q_{x+k} = 9.98756$, we get $b = 200$.