EXAM FM QUESTIONS OF THE WEEK

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Week of January 23/06

A mortgage loan has level monthly payments for 30 years, with the first payment due one month after the loan is made. It is found that the total principal repaid in the first 20 years of the loan is 80% of the principal repaid in the final 10 years of the loan. Find the loan interest rate as a nominal annual rate compounded monthly.

The solution can be found below.

Week of January 23/06 - Solution

The loan will have 360 monthly payments.
With monthly interest rate j and monthly payment K, the principal paid in the t-th payment is
$PR_t = Kv_j^{360-t+1} = Kv_j^{361-t}$, for $t = 1, 2, ..., 360$

The principal repaid in the first 20 years is
$K[v_j^{360} + v_j^{359} + \cdots + v_j^{241} + v_j^{240} + v_j^{239} + \cdots + v_j^{121}]$
$= K(v^{240} + v^{120})[v_j^{120} + v_j^{119} + \cdots + v_j^{1}]$.

The principal repaid in the final 10 years is $K[v_j^{120} + v_j^{119} + \cdots + v_j^{1}]$.

We are told that
$K(v^{240} + v^{120})[v_j^{120} + v_j^{119} + \cdots + v_j^{1}] = .8K[v_j^{120} + v_j^{119} + \cdots + v_j^{1}]$,
from which it follows that $(v^{240} + v^{120}) = .8$.

We use the substitution $A = v_j^{120}$, and the equation becomes $A + A^2 = .8$.
Solving this quadratic equation for A results in $A = .524695$
(or $A = -1.524695$, but we ignore the negative root for $A = v_j^{120}$).
Then $v_j^{120} = .524695$, and $v_j = .994640$, and $j = .005389$ is the monthly loan interest rate.
Finally, $i^{(12)} = 12j = .0647$.