EXAM C QUESTIONS OF THE WEEK

S. Broverman, 2005

Week of January 23/06

A Pareto distribution is know to have a value of $\theta = 100$. A random sample of 100 observations is taken from the distribution. 30 of the observations were right-censored at 50 (all 30 values are over 50, but the actual values are not known). The maximum likelihood estimate of α is 3.50 . Suppose that of the 30 data points censored at 50, 25 of them are greater than 60, and the others are known to be 52 , 55 , 56 , 56 and 58 . Using this information, find the revised mle of α.

Solution can be found below.

Week of January 23/06 - Solution

Based on the original data, the mle of α is $\widehat{\alpha} = \frac{n}{\sum_1^n ln(\frac{x_i+\theta}{\theta}) + n_1\, ln(\frac{u+\theta}{\theta})}$,

where $n = 70$ is the number of non-censored data points, the x_i's are the uncensored sample values, $n_1 = 30$ and $u = 50$. Therefore $3.5 = \frac{70}{\sum_1^{70} ln(\frac{x_i+\theta}{\theta}) + 30\, ln(\frac{150}{100})}$,

from which we get $\sum_1^{70} ln(\frac{x_i+\theta}{\theta}) = 7.8360$.

Using the revised data, there are now 75 uncensored observations, and the mle is

$$\widehat{\alpha} = \frac{75}{\sum_1^{75} ln(\frac{x_i+\theta}{\theta}) + 25\, ln(\frac{160}{100})} = \frac{75}{7.8360 + ln(\frac{152}{100}) + ln(\frac{155}{100}) + ln(\frac{156}{100}) + ln(\frac{156}{100}) + ln(\frac{158}{100}) + 25\, ln(\frac{160}{100})} = 3.442 \;.$$