EXAM P QUESTIONS OF THE WEEK

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Week of January 16/06

An econometric study of the population of the island of Falkvinas has found income disparities between those who have a university degree and those who do not. A Falkvinian who is chosen at random from all Falkvinians with a university degree has an annual income that is normally distributed with a mean of 80,000 BA (the Falkvinian currency is the BritArg) and a standard deviation of 20,000 BA. A Falkvinian who is chosen at random from all Falkvinians without a university degree has an annual income that is normally distributed with a mean of 100,000 BA (the Falkvinian currency is the BritArg) and a standard deviation of 40,000 BA. Suppose that Cedric is a randomly chosen Falkvinian with a university degree and Juan is a randomly chosen Falkvinian without a university degree. Find the probability that Juan's annual income is at least 30,000 BA greater than Cedric's annual income.

A) .35 B) .37 C) .39 D) .41 E) .43

The solution can be found below.

Week of January 16/06 - Solution

Suppose that Juan's annual income is X and Cedric's is Y. Since they were randomly chosen, X and Y are independent. We wish to find the probability $P[X > Y + 30,000]$, which is the same as $P[X - Y > 30,000]$.

Since X and Y are both normally distributed, so is $X - Y$.
The mean of $X - Y$ is $E[X] - E[Y] = 100,000 - 80,000 = 20,000,$
and since X and Y are independent, the variance of $X - Y$ is
$Var[X] + Var[Y] = 20,000^2 + 40,000^2 = 2,000,000,000$.

Then $Z = \frac{(X-Y)-20,000}{\sqrt{60,000}}$ has a standard normal distribution, and
$P[X - Y > 30,000] = P[\frac{X-Y-20,000}{\sqrt{2,000,000,000}} > \frac{30,000-20,000}{\sqrt{2,000,000,000}}]$
$= P[Z > .223] = 1 - \Phi(.223)$.
If we round to 2 decimals, we get $1 - \Phi(.22) = 1 - .59 = .41$.

This is the value from the Exam P table for the standard normal distribution. We could have applied linear interpolation in the normal table between $\Phi(.22) = .5871$ and $\Phi(.23) = .5910$ to approximate $\Phi(.223)$. Whether or not this is necessary would depend on the accuracy implied in the answers. In this problem, answers were to 2 decimal places. Initial calculations should be to more than 2 decimals, but then round to 2 decimals should be done at the end. Interpolation would result in an answer of .41 also.