EXAM M QUESTIONS OF THE WEEK

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Week of January 16/06

A 3-decrement model for mortality for individuals with a particular genetic makeup identifies three causes of death. The three causes are Disease A, Disease B and all other causes. According to the model, at age x the following absolute rates of decrement are known:
$q_x'^{(A)} = .2$ (Disease A) , $q_x'^{(B)} = .3$ (Disease B) , $q_x'^{(C)} = .1$ (all other causes) .
As a result of ongoing research, Disease B is reclassified as two separate diseases, Disease B_1 and Disease B_2 . The multiple decrement probabilities for original Disease B and the reclassified Diseases B_1 and B_2 are consistent in that $q_x^{(B)}$ in the original 3 decrement model is equal to the sum of $q_x^{(B_1)}$ and $q_x^{(B_2)}$ in the reclassification model. Furthermore, it is determined that Disease B_1 is twice as prevalent as Disease B_2 for individuals with the particular genetic makeup being studied. Find the values of $q_x'^{(B_1)}$ and $q_x'^{(B_2)}$ for the reclassification model if we assume UDD in the multiple decrement model is assumed after reclassification.

The solution can be found below.

Week of January 16/06 - Solution

For the 3 decrement model, we have $p_x^{(\tau)} = p_x'^{(A)} \cdot p_x'^{(B)} \cdot p_x'^{(C)} = (.8)(.7)(.9) = .504$.

For the reclassification model, we have $p_x^{(\tau)} = p_x'^{(A)} \cdot p_x'^{(B_1)} \cdot p_x'^{(B_2)} \cdot p_x'^{(C)}$.
The survival probability is not affected by the reclassification, so that
$.504 = p_x'^{(A)} \cdot p_x'^{(B_1)} \cdot p_x'^{(B_2)} \cdot p_x'^{(C)} = (.8) \cdot p_x'^{(B_1)} \cdot p_x'^{(B_2)} \cdot (.9)$.
It follows that $p_x'^{(B_1)} \cdot p_x'^{(B_2)} = .7$.

We are given that $q_x^{(B_1)} = 2 \cdot q_x^{(B_2)}$.

Under UDD in the multiple table, $q_x^{(B_1)} = \frac{ln\, p_x'^{(B_1)}}{ln\, p_x^{(\tau)}} \cdot q_x^{(\tau)}$ and $q_x^{(B_2)} = \frac{ln\, p_x'^{(B_2)}}{ln\, p_x^{(\tau)}} \cdot q_x^{(\tau)}$.
It follows that $2 = \frac{q_x^{(B_1)}}{q_x^{(B_2)}} = \frac{ln\, p_x'^{(B_1)}}{ln\, p_x'^{(B_2)}}$, so that $ln\, p_x'^{(B_1)} = 2 \cdot ln\, p_x'^{(B_2)} = ln\, [\, (p_x'^{(B_2)})^2]$.
Then, from $p_x'^{(B_1)} \cdot p_x'^{(B_2)} = .7$, we get $(p_x'^{(B_2)})^2 \cdot p_x'^{(B_2)} = .7$, and $p_x'^{(B_2)} = (.7)^{1/3}$,
and $p_x'^{(B_1)} = (.7)^{2/3}$.
Finally, $q_x'^{(B_1)} = 1 - (.7)^{2/3} = .212$ and $q_x'^{(B_2)} = 1 - (.7)^{1/3} = .112$.