

## EXAM FM QUESTIONS OF THE WEEK

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### Week of January 16/06

On January 1, 2006 a loan of \$10,000 is made. The loan is to be repaid over 10 years with level monthly payments starting February 1, 2006. The annual effective rate of interest on the loan is 7%. The borrower can choose to repay the loan in full at any time during the 10 year loan period, but there is a prepayment penalty. If the loan is prepaid on a scheduled payment date, the prepayment penalty is 10% of what the outstanding balance would be just after that regularly scheduled payment.

The borrower establishes a savings account to which annual payments of \$P are made. The savings account earns 9% per year, compounded monthly. The first payment to the savings account is to be made on December 31, 2006. Just prior to the 53rd scheduled loan repayment the savings account balance is sufficient to pay both the outstanding loan amount and prepayment penalty. Find P.

**The solution can be found below.**

## **Week of January 16/06 - Solution**

The equivalent monthly loan rate is interest is  $j = (1.07)^{1/12} - 1 = .005654$ .

The scheduled loan payment is  $\frac{10,000}{a_{\overline{120}|j}} = 115.00$ .

The outstanding balance just after the 53rd payment would be  $115a_{\overline{67}|j} = 6,399$ , so the payment needed (before penalty) to repay the balance plus 53rd payment is  $6,399 + 115 = 6514$ . The prepayment penalty is 10% of 6,399 (which is 640), so the total needed to repay the loan at that time is  $6,399 + 115 + 640 = 7,154$ .

The 53rd payment occurs 4 years and 5 months after the loan is issued. The monthly rate of interest on the savings account is  $\frac{.09}{12} = .0075$ , and the equivalent annual effective interest rate on the savings account is  $i = (1.0075)^{12} - 1 = .0938$ .

The balance in the savings account at the end of the 4th year is  $Pa_{\overline{4}|i}$ ,

and the balance at the time of the 53rd loan payment would be  $Pa_{\overline{4}|i}(1.0075)^5 = 4.774P$  (5 months after the end of 4 years).

In order for this to be enough to pay off the loan, we must have  $4.774P = 7,154$ , from which we get  $P = 1,499$ .