EXAM C QUESTIONS OF THE WEEK

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Week of January 16/06

X has a (3-parameter) beta distribution with unknown parameter a, and known parameters $b = 2$, $\theta = 1$. For a random sample of 10 observations, $x_1, \ldots, x_{10}$, it is found that
$\sum_{i=1}^{10} ln\, x_i = -12$.
Find the maximum likelihood estimate of a.

Solution can be found below.

Week of January 16/06 - Solution

$f(x) = \frac{\Gamma(a+b)}{\Gamma(a)\cdot\Gamma(b)} \cdot x^{a-1}(1-x)^{b-1} = \frac{\Gamma(a+2)}{\Gamma(a)\cdot\Gamma(2)} \cdot x^{a-1}(1-x)$ for $0 < x < 1$ ($\theta = 1$) .

This can be written as $f(x) = (a+1)\cdot a \cdot x^{a-1}(1-x)$, and the log of the density is $ln\, f(x) = ln(a+1) + ln\, a + (a-1)\, ln\, x + ln(1-x)$.

The derivative with respect to a of the log of the density is $\frac{d}{da}\, ln\, f(x) = \frac{1}{a+1} + \frac{1}{a} + ln\, x$.

The derivative of the loglikelihood for the sample is
$\frac{d}{da}\ell = \sum_{i=1}^{10} \frac{d}{da}\, ln\, f(x_i) = \frac{10}{a+1} + \frac{10}{a} + \sum_{i=1}^{10} ln\, x_i = \frac{10}{a+1} + \frac{10}{a} - 12$.

The mle of a is found by setting this equal to 0 and solving for a: $\frac{10}{a+1} + \frac{10}{a} - 12 = 0$.

This equation can be rewritten as the quadratic equation $12a^2 - 8a - 10 = 0$.
The roots of the equation are $a = 1.305$ and $-.638$. We discard the negative root.