EXAM FM QUESTIONS OF THE WEEK

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Week of July 30/07

Smith receives payments at the end of each month from an income trust.
The first payment is $1000 and Smith receives it in January of 2007.
The monthly payments during each calendar year are level, but each January, there is a 3% increase in the amount of the monthly payment for the new year.
On the last day of each month, Smith deposits the payment into an account earning a nominal annual rate of interest of 6% compounded monthly.
Find the calendar year during which Smith's account balance first reaches $250,000.

The solution can be found below.

Week of July 30/07 - Solution

At the end of 2007, after the 12th deposit has been made and interest has been credited, Smith's account balance is $1000 s_{\overline{12}|.005} = 12,335.56$. If we regard Smith's deposits as being equivalent to annual deposits at the end of each year, then the equivalent deposit amounts will be 12,335,56 in 2007 , $12,335,56(1.03)$ in 2008 , $12,335.56(1.03)^2$ in 2009, etc.
On an equivalent annual deposit basis, Smith's deposits form a geometrically increasing annuity. The annual effective interest rate is $(1.005)^{12} - 1 = .061678$ and the geometric increase of 3% per year.

The accumulated value at the end of the n-th year is $12,335,56 \cdot \frac{(1.061678)^n - (1.03)^n}{.061678 - .03}$.
We wish to find the smallest n for which this balance is at least 250,000 .

$$12,335,56 \cdot \frac{(1.061678)^n - (1.03)^n}{.061678 - .03} \geq 250,000 \rightarrow (1.061678)^n - (1.03)^n \geq .642 \text{ .}$$

This inequality cannot be solved algebraically. By trial and error, we have
$(1.061678)^{10} - (1.03)^{10} = .475 < .642$,
$(1.061678)^{20} - (1.03)^{20} = 1.76 > .642$,
$(1.061678)^{15} - (1.03)^{15} = .896 > .642$,
$(1.061678)^{12} - (1.03)^{12} = .625 < .642$,
$(1.061678)^{13} - (1.03)^{13} = .709 > .642$.

The balance of 250,000 is reached sometime during the 13th year. That is calendar year 2019.