EXAM FM QUESTIONS OF THE WEEK

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Week of January 21/08

Smith purchases a perpetuity immediate with monthly payments which has a first payment of 1000. Each subsequent payment is 10 larger than the previous payment. The interest rate is a nominal annual rate of 12% compounded monthly.

Jones wishes to arrange an inflation-indexed perpetuity immediate that has the same unindexed payments as Smith's perpetuity, but Jones wants his perpetuity to have a monthly index of $\frac{1}{2}$%. This means that if Smith's sequence of payments is $C_1, C_2, C_3, \ldots$, then the payments in Jones's perpetuity are $(1.005)C_1, (1.005)^2, (1.005)^3 C_3, \ldots$. The interest rate on Jones's perpetuity is the same as Smith's. Find the ratio of the present value of Jones's perpetuity to that of Smith's.

The solution can be found below.

Week of January 21/08 - Solution

The present value of Smith's perpetuity is
$1000v_{.01} + 1010v_{.01}^2 + 1020v_{.01}^3 + \cdots = 1000a_{\overline{\infty}|.01} + 10v_{.01}(Ia_{\overline{\infty}|.01})$
$= 1000 \times \frac{1}{.01} + 10v_{.01} \times (\frac{1}{.01} + \frac{1}{.01^2}) = 200,000$.

The present value of Jones's perpetuity is
$1000v(1.005) + 1010v^2(1.005)^2 + 1020v^3(1.005)^3 + \cdots$
$= 1000(1.005)v[1 + (1.005v) + (1.005)^2v^3 + \cdots]$
$+ 10v^2(1.005)^2[1 + 2(1.005v) + 3(1.005v)^2 + \cdots]$

Using the infinite geometric series, we have
$1000(1.005)v[1 + (1.005v) + (1.005)^2v^3 + \cdots]$
$= 1000(1.005)v \times \frac{1}{1-1.005v} = 1000(1.005)(\frac{1}{1.01}) \times \frac{1}{1-\frac{1.005}{1.01}} = 201,000$.

The infinite increasing geometric series formula is $1 + 2a + 3a^2 + \cdots = \frac{1}{(1-a)^2}$.
With $a = 1.005v$ we get
$10v^2(1.005)^2[1 + 2(1.005v) + 3(1.005v)^2 + \cdots]$
$= 10v^2(1.005)^2 \times \frac{1}{(1-1.005v)^2} = 404,010$.

The total pv of Jones's perpetuity is $605,010$.
The ratio of Jones's pv to Smith's pv is $\frac{605,010}{200,000} = 3.025$.