EXAM FM QUESTIONS OF THE WEEK

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Week of February 11/08

A bond with face amount 100 and with annual coupons matures n years from now (n is an integer). The price of the bond is 116.38.

A second bond with face amount 100 and with annual coupons matures $n+1$ years from now, and has a coupon rate that is equal to 1% plus the first bond's coupon rate.
The price of the second bond is 129.73.

Both bonds are priced at an annual effective yield to maturity of 5.2%.

Find the coupon rate and time to maturity of each bond.

The solution can be found below.

Week of February 11/08 - Solution

$P_1 = 100 + 100(r_1 - j)\, a_{\overline{n}|j} = 116.38$
and
$P_2 = 100 + 100(r_2 - j)\, a_{\overline{n+1}|j} = 100 + 100(r_2 - j)\, a_{\overline{n}|j} + 100(r_2 - j)v^{n+1} = 129.73$.

Subtracting the first from the second results in.
$P_2 - P_1 = 13.35 = 100(r_2 - r_1)\, a_{\overline{n}|j} + 100(r_2 - j)v^{n+1} = a_{\overline{n}|j} + 100(r_2 - j)v^{n+1}$.

Then from $P_2 = 129.73 = 100 + 100(r_2 - j)\, a_{\overline{n+1}|j}$,
we get $100(r_2 - j) = \frac{29.73}{a_{\overline{n+1}|j}}$, and substituting this into
$13.35 = a_{\overline{n}|j} + 100(r_2 - j)v^{n+1}$ results in
$13.35 = a_{\overline{n}|j} + \frac{29.73}{a_{\overline{n+1}|j}} \cdot v^{n+1} = a_{\overline{n}|j} + \frac{29.73}{s_{\overline{n+1}|j}}$
$= a_{\overline{n}|j} + 29.73(\frac{1}{a_{\overline{n+1}|j}} - j)$.
Using the identity $a_{\overline{n}|j} = (1+i)a_{\overline{n+1}|j} - 1$, we get
$13.35 = (1+i)a_{\overline{n+1}|j} - 1 + 29.73(\frac{1}{a_{\overline{n+1}|j}} - j)$
$= (1.052)X - 1 + 29.73(\frac{1}{X} - .052) = 13.35$, where $X = a_{\overline{n+1}|j}$.

This can be formulated as a quadratic equation $1.052X^2 - 15.896X + 29.73 = 0$.

The roots of the equation are $X = 12.924$ or 2.187 .
Since $X = a_{\overline{n+1}|.052}$, if $a_{\overline{n+1}|.052} = 12.924$, we have $n + 1 = 22$,
and if $a_{\overline{n+1}|.052} = 2.187$, we get $n = 2.4$, which is not an integer.

Therefore, $n = 21$ is the time to maturity for the first bond and the second bond matures in 22 years. From $116.38 = 100 + 100(r_1 - j)\, a_{\overline{n}|j}$ we get $r_1 = .065$,
and from $129.73 = 100 + 100(r_2 - j)\, a_{\overline{n+1}|j}$, we get $r_2 = 075$.