EXAM FM QUESTIONS OF THE WEEK

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Week of December 31/07

At time 0, Smith makes a deposit of X into an account earning annual effective rate of interest i, and Jones makes a deposit of X into an account earning nominal annual interest rate $2j$ compounded semi-annually. The amount of interest earned in Smith's account in year n is the same as the amount of interest earned in Jones' account. Which of the following is true?
I. $i < 2j$ II. $i = 2j$ III. $i > 2j$

The solution can be found below.

Week of December 13/07 - Solution

The amount of interest in the nth year in Smith's account is $(1+i)^n \times i$.

The amount of interest in the nth year in Jones' account is $(1+j)^{2n} \times [(1+j)^2 - 1]$.

We are given that $(1+i)^n \times i = (1+j)^{2n} \times [(1+j)^2 - 1]$.

If $i \leq 2j$, then $1 + i = 1 + 2j < 1 + 2j + j^2 = (1+j)^2$,
and $i < 2j + j^2 \leq (1+j)^2 - 1$. It follows that
$(1+i)^n < (1+j)^{2n}$ and $i < (1+j)^2 - 1$, so that
$(1+i)^n \times i < (1+j)^{2n} \times [(1+j)^2 - 1]$.

Therefore, it must be true that $i > 2j$.