

EXAM FM QUESTIONS OF THE WEEK

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Week of August 27/07

On September 1, 2006, Smith obtained a mortgage loan of \$400,000 at a nominal annual interest rate of 3% compounded monthly. The loan has an amortization period of 30 years, with monthly payments on the last day of each month. Just after the payment on August 1, 2007, Smith must refinance the loan balance at an interest rate of 6% compounded monthly. The monthly payment on the refinanced loan is too high for Smith. The lender doesn't want to foreclose on Smith's loan, and allows Smith to make monthly payments equal to 90% of the interest due each month. Assuming that this arrangement continues for some time, find the date on which the outstanding balance reaches \$400,000. This will happen at a month-end, just before a payment is made.

The solution can be found below.

Week of August 27/07 - Solution

Smith's initial monthly payment is $\frac{400,000}{a_{360|.0025}} = 1686.42$.

Outstanding balance on August 31, 2007 after the 12th payment is

$$1686.42a_{\overline{348}|.0025} = 391,649.$$

Suppose that the monthly interest rate is j starting on September 1, 2007.

Since Smith pays 90% of the interest at the end of each month, the balance on Sept. 30, 2007 after the payment will be

$$391,649(1 + j) - (.9)391,649j = 391,649(1 + .1j).$$

This pattern continues each month. The balance outstanding after m months of this arrangement will be $391,649(1 + .1j)^m$.

The balance at the end of m months, just before the monthly payment is made is $391,649(1 + .1j)^{m-1}(1 + j)$.

Since $j = .005$ is the monthly interest rate starting on Sept. 1, 2007, the balance m months after August 31, 2007, but just before the monthly payment is made, is $391,649(1.0005)^{m-1}(1.005)$.

This will reach 400,000 if $391,649(1.0005)^{m-1}(1.005) = 400,000$.

Solving for m results in 33.23.

33 months after August 31, 2007 is May 31, 2010.

The balance, after the monthly payment on that date will be

$$391,649(1.0005)^{33} = 398,163.$$

On June 30, 2010, just before the monthly payment, the balance will be

$$398,163(1.005) = 400,154.$$