

EXAM FM QUESTION OF THE WEEK

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Week of April 21/08

A non-dividend paying stock has current price S_0 .

The continuously compounded risk free rate of interest is 6%.

The maximum loss at expiry on a European floor with a strike price of \$32 expiring in 6 months is 1.91.

The maximum profit at expiry on a European cap with a strike price of \$32 expiring in 6 months is 29.04.

Find the combined price at time 0 of a European put and call expiring in 6 months with a strike price of 32.

The solution can be found below.

Week of April 21/08 - Solution

A combination of being in a long position on an asset and having a purchased put option on the asset is called a floor.

The profit at expiry on the floor is
$$\begin{cases} K - (S_0 + P_0)e^{rT} & \text{if } S_T \leq K \\ S_T - (S_0 + P_0)e^{rT} & \text{if } S_T > K \end{cases},$$

so the maximum loss at expiry (negative of the minimum profit) at expiry is $(S_0 + P_0)e^{rT} - K$.

Therefore, $(S_0 + P_0)e^{0.03} - 32 = 1.91$, so that $(S_0 + P_0)e^{0.03} = 33.91$.

A cap is the combination of (i) a short position in the asset, along with (ii) a purchased call option in the asset.

The profit at expiry on the cap combination is
$$\begin{cases} (S_0 - C_0)e^{rT} - S_T & \text{if } S_T \leq K \\ (S_0 - C_0)e^{rT} - K & \text{if } S_T > K \end{cases},$$

so the maximum profit at expiry occurs at the minimum value of S_T , which is 0, so the maximum profit at expiry is $(S_0 - C_0)e^{rT}$. Therefore $(S_0 - C_0)e^{0.03} = 29.04$.

Then $(S_0 + P_0)e^{0.03} - (S_0 - C_0)e^{0.03} = (P_0 + C_0)e^{0.03} = 33.91 - 29.04 = 4.87$.

Then, $P_0 + C_0 = 4.87e^{-0.03} = 4.73$.