EXAM P QUESTIONS OF THE WEEK

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Week of February 6/06

X is a continuous random variable with the pdf

$f_X(x) = cx^{-4}$ on the interval (a, ∞) , where $a > 0$.

Y is a random variable defined by $Y = \frac{1}{X}$.

Find $E(X) \cdot E(Y)$.

The solution can be found below.

Week of February 6/06 - Solution

Since $Y = X^{-1}$, it follows that $X = Y^{-1} = k(Y)$.

The pdf of Y can be found from the formula

$$f_Y(y) \;=\; f_X(k(y)) \cdot |k'(y)| \;=\; c(y^{-1})^{-4} \cdot |-y^{-2}| = cy^2 \text{ .}$$

Since X is defined on the interval (a, ∞) , the interval for Y is $(0, \frac{1}{a})$.

X must be a pdf, and therefore, $\int_a^\infty f_X(x)\,dx = \int_a^\infty cx^{-4}\,dx = \frac{c}{3a^3} = 1$,
and therefore $c = 3a^3$, and $f_X(x) = 3a^3x^{-4}$ on the interval (a, ∞) .

The mean of X is $E(X) = \int_a^\infty x \cdot f_X(x)\,dx = \int_a^\infty x \cdot 3a^3x^{-4}\,dx = \frac{3a}{2}$.

The pdf of Y is $f_Y(y) = cy^2 = 3a^3y^2$ on the interval $(0, \frac{1}{a})$.

The mean of Y is $E(Y) = \int_0^{1/a} y \cdot 3a^3y^2\;dy = \frac{3}{4a}$.

Then, $E(X) \cdot E(Y) = \frac{9}{8}$.