EXAM M QUESTIONS OF THE WEEK

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Week of February 6/06

A Poisson distribution has mean 1 and probability function

$f_k = \frac{e^{-1}}{k!}$ for $k = 0, 1, 2, ...$

A geometric distribution has mean $\frac{1-p}{p}$, where $0 < p < 1$, and probability function

$g_k = p(1-p)^k$ for $k = 0, 1, 2, ...$

A comparison of the two distributions is made by summing the ratios of the probabilities $\sum_{k=0}^{\infty} \frac{f_k}{g_k}$.

Formulate that summation as a function of p, and find the value of p that minimizes the sum.

The solution can be found below.

Week of February 6/06 - Solution

$$\frac{f_k}{g_k} = \left(\frac{e^{-1}}{k!}\right)\Big/\left(p(1-p)^k\right) = \frac{e^{-1}(1-p)^k}{p \cdot k!}$$

$$\sum_{k=0}^{\infty} \frac{f_k}{g_k} = \frac{e^{-1}}{p} \cdot \sum_{k=0}^{\infty} \frac{1/(1-p)^k}{\cdot k!} = \frac{e^{-1}}{p} \cdot e^{1/(1-p)} = \frac{e^{p/(1-p)}}{p} \ .$$

It can be seen that $\frac{e^{p/(1-p)}}{p}$ approaches ∞ as $p \to 0^+$ (from above 0) and as $p \to 1^-$ (from below 1).

To find where $\frac{e^{p/(1-p)}}{p}$ is minimized, we take $ln[\frac{e^{p/(1-p)}}{p}]$, and minimize that.

$ln[\frac{e^{p/(1-p)}}{p}] = \frac{p}{1-p} - ln\, p$, and $\frac{d}{dp}\ ln[\frac{e^{p/(1-p)}}{p}] = \frac{d}{dp}\frac{p}{1-p} - ln\, p$

$$= \frac{1}{(1-p)^2} - \frac{1}{p} = \frac{p-(1-p)^2}{(1-p)^2 \cdot p} \ .$$

The critical points occur where $p-(1-p)^2 = -1+3p-p^2 = 0$,
so that $p = \frac{3 \pm \sqrt{5}}{2}$. We ignore the root > 1 , and $p = \frac{3-\sqrt{5}}{2} = .3820$.