EXAM C QUESTIONS OF THE WEEK

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Week of February 6/06

Random sampling from the distribution of X results in the three sample values 1, 2 and 4.

A uniform distribution on the interval $[0, \theta]$ is fitted to the data set by finding the value of θ that minimizes the Kolmogorov-Smirnov goodness-of-fit statistic.

Find θ.

Solution can be found below.

Week of February 6/06 - Solution

The empirical cdf is $F_3(1) = \frac{1}{3}$, $F_3(2) = \frac{2}{3}$ and $F_3(4) = 1$.

For parameter value θ, the K-S statistic is the maximum of

$|\frac{1}{\theta}|$, $|\frac{1}{3} - \frac{1}{\theta}|$, $|\frac{2}{\theta} - \frac{1}{3}|$, $|\frac{2}{3} - \frac{2}{\theta}|$, $|\frac{4}{\theta} - \frac{2}{3}|$, $|1 - \frac{4}{\theta}|$.

Since there is an observed value of 4, θ must be at least 4.
With $\theta = 4$, we see that the K-S statistic is the maximum of
$|\frac{1}{4}|$, $|\frac{1}{3} - \frac{1}{4}|$, $|\frac{2}{4} - \frac{1}{3}|$, $|\frac{2}{3} - \frac{2}{4}|$, $|\frac{4}{4} - \frac{2}{3}|$, $|1 - \frac{4}{4}|$.
This maximum is $\frac{1}{3}$.

If $\theta > 6$, then $|1 - \frac{4}{\theta}| = 1 - \frac{4}{\theta} > \frac{1}{3}$, so that to minimize the K-S statistic, we want $\theta \leq 6$.

Therefore $4 \leq \theta \leq 6$, and the six quantities are
$\frac{1}{\theta}$, $\frac{1}{3} - \frac{1}{\theta}$, $\frac{2}{\theta} - \frac{1}{3}$, $\frac{2}{3} - \frac{2}{\theta}$, $\frac{4}{\theta} - \frac{2}{3}$, $1 - \frac{4}{\theta}$ (the absolute value signs can be removed because $4 \leq \theta \leq 6$.

To find the K-S statistic for a given θ, we must find the maximum of the six quantities above. Suppose that we pick a value of θ for which $\frac{1}{3} - \frac{1}{\theta} = \frac{2}{\theta} - \frac{1}{3}$ (the 2nd set equal to the third). Solving for θ results in $\theta = \frac{9}{2}$. For that value of θ we know that the K-S statistic is at least $\frac{1}{9}$ (but might be larger when the other quantities are calculated). Any other value of θ will definitely result in a larger K-S statistic than $\frac{1}{9}$.
For each "crossover" value of θ (a value of θ that is found by setting two of the 6 quantities equal), we find the K-S statistic, The minimum possible K-S statistic occurs at the minimum of these.

The "crossover" values of θ are 4 , 4.5 , 4.8 , 5 and 6 .
The K-S statistic in each of these cases are $\frac{1}{3}$, $\frac{2}{9}$, $\frac{1}{4}$, $\frac{4}{15}$, $\frac{1}{3}$.
The minimum of these is $\frac{2}{9}$ and it occurs when $\theta = 4.5$.