

EXAM C QUESTIONS OF THE WEEK

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Week of February 27/06

A Cox proportional hazards model is applied to model the future mortality of non-smokers and smokers who are currently 50 years old. The model assumes that non-smoker mortality follows a non-parametric baseline distribution for time until death from age 50. There is a single covariate, Z , and $Z = 0$ indicates a non-smoker, and $Z = 1$ indicates a smoker. Age of death is available for 10 individuals now at age 50, 5 of whom are smokers and 5 of whom are non-smokers. The ages of death of the smokers are 55, 58, 62, 67, 77, and the ages of death of the non-smokers are 58, 63, 72, 81, 91. The partial maximum likelihood estimate of β is $\hat{\beta} = 1.426$.

Using Breslow's approach, find the estimated probability that a 50 year old smoker will survive to at least age 75.

Solution can be found below.

Week of February 27/06 - Solution

We first use Breslow's approach to estimate the baseline cumulative hazard function at $t = 25$, $\hat{H}_0(25)$.

The death times up to age 75 (time 25 as measured from age 50) are 5 (smoker), 8 (smoker and non-smoker), 12 (smoker), 13 (non-smoker), 17 (smoker) and 22 (non-smoker). The Breslow estimate is

$$\hat{H}_0(25) = \frac{1}{5+5e^{\hat{\beta}}} + \frac{2}{5+4e^{\hat{\beta}}} + \frac{1}{4+3e^{\hat{\beta}}} + \frac{1}{4+2e^{\hat{\beta}}} + \frac{1}{3+2e^{\hat{\beta}}} + \frac{1}{3+e^{\hat{\beta}}} = .501 .$$

The cumulative hazard function at $t = 25$ for a smoker is

$$\hat{H}_s(25) = e^{\hat{\beta}} \cdot \hat{H}_0(25) = e^{1.426} \cdot (.501) = 2.09 , \text{ and the Breslow estimated survival probability to age 75 from age 50 for a smoker is } \hat{S}_s(25) = e^{-\hat{H}_s(25)} = e^{-2.09} = .124 .$$