

EXAM P QUESTIONS OF THE WEEK

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Week of February 20/06

A town's maintenance department has estimated that the cost of snow removal after a major snowstorm is \$100,000. Historical information suggests that the number of major snowstorms in a winter season follows a geometric distribution for which the probability of no major snowstorms in a season is .4. The town purchases an insurance policy which pays nothing if there is one or less major snowstorms in the season, but the insurance pays 50% of all seasonal snow removal costs for major snowstorms if there are 2 or more major snowstorms. Find the expected payout by the insurer.

The solution can be found below.

Week of February 20/06 - Solution

For a geometrically distributed random variable with $P(N = 0) = p = .4$, the probability function is $P(N = k) = p(1 - p)^k = (.4)(.6)^k$, and the mean is $E[N] = \frac{1-p}{p} = \frac{.6}{.4} = 1.5$.

The insurance pays 50% of all seasonal snow removal costs for major snowstorms if there are 2 or more major snowstorms. The expected amount paid by the insurance is $50,000 \times [2 \cdot P(N = 2) + 3 \cdot P(N = 3) + \dots]$.

From the definition of $E[N]$, we have

$$\begin{aligned} 1.5 &= E[N] = 0 \cdot P(N = 0) + 1 \cdot P(N = 1) + 2 \cdot P(N = 2) + 3 \cdot P(N = 3) + \dots \\ &= P(N = 1) + [2 \cdot P(N = 2) + 3 \cdot P(N = 3) + \dots] \\ &= .24 + [2 \cdot P(N = 2) + 3 \cdot P(N = 3) + \dots] \end{aligned}$$

It follows that $2 \cdot P(N = 2) + 3 \cdot P(N = 3) + \dots = 1.5 - .24 = 1.26$.

The expected amount paid by the insurer is $50,000(1.26) = 63,000$.