EXAM FM QUESTIONS OF THE WEEK

S. Broverman, 2006

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A real-return bond has payments linked to a specified inflation measure (such as the consumer price index). Each coupon payment is the quoted coupon payment amount multiplied by the accumulated inflation since the date the bond was issued. Also, the redemption amount is the face amount stated at the time the bond is issued multiplied by accumulated inflation since the date the bond was issued.

Suppose that a real return bond is issued with n coupon periods until maturity. The coupon rate is r per period, the maturity amount is 1 on the issue date and the assumed inflation rate is f per coupon period (compounding every coupon period). Assume that present value is found using a yield rate of j per coupon period. Find an expression for the present value of the bond on the issue date by expressing the present value of the coupons as a geometrically increasing annuity and adding the present value of the maturity amount. Show that the bond price is equal to a regular (not real-return) bond with level coupons at rate r, and a maturity amount of 1 valued at the yield rate $k = \frac{j-f}{1+f}$ per coupon period.

The solution can be found below.

Week of February 13/06 - Solution

The after-inflation bond payments are

Time: 1 2 $n-1$ n

Pmt: $r(1+f)$ $r(1+f)^2$ $\ldots$ $r(1+f)^{n-1}$ $r(1+f)^n + (1+f)^n$

(the payment at time n is after-inflation coupon plus after-inflation maturity payment).

The present value of coupons is

$$r[\tfrac{1+f}{1+j} + (\tfrac{1+f}{1+j})^2 + \cdots (\tfrac{1+f}{1+j})^n] = r(\tfrac{1+f}{1+j})[\tfrac{1-(\frac{1+f}{1+j})^n}{1-\frac{1+f}{1+j}}] = r[\tfrac{1-(\frac{1+f}{1+j})^n}{\frac{j-f}{1+j}}]$$

and the present value of the maturity amount is $(\tfrac{1+f}{1+j})^n$,

so the total bond price is $r[\tfrac{1-(\frac{1+f}{1+j})^n}{\frac{j-f}{1+j}}] + (\tfrac{1+f}{1+j})^n$.

With yield rate $k = \frac{j-f}{1+f}$ the present value factor is $v_k = \frac{1}{1+k} = \frac{1+f}{1+j}$.

and for a bond with level coupons and maturity amount 1, the bond price is

$$r \cdot a_{\overline{n}|k} + v_k^n = r[\tfrac{1}{1+k} + (\tfrac{1}{1+k})^2 + \cdots + (\tfrac{1}{1+k})^n] + (\tfrac{1}{1+k})^n$$
$$= r[\tfrac{1+f}{1+j} + (\tfrac{1+f}{1+j})^2 + \cdots (\tfrac{1+f}{1+j})^n] + (\tfrac{1+f}{1+j})^n .$$

This is the same as the first bond price formulation.