EXAM C QUESTIONS OF THE WEEK

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Week of February 13/06

A sample of size n is used to estimate the parameters in two possible models for the data.
The maximized log-likelihood for the 3-parameter generalized Pareto model is ℓ_A , and the maximized log-likelihood for the exponential model is ℓ_B. You are given that according to the Schwarz Bayesian Criterion, model A is preferred to model B. You are also given that according to the likelihood ratio test, in which the null hypothesis is that model B is acceptable, and the alternative hypothesis is that model A is preferable to model B, the null hypothesis is rejected at the 5% level of significance but not at the 1% level of significance. Find the maximum value of n that is compatible with these results.

Solution can be found below.

Week of February 13/06 - Solution

The Schwarz Bayesian Criterion compares $\ell_A - \frac{3}{2}\ln(n)$ and $\ell_B - \frac{1}{2}\ln(n)$,
and since model A is preferable to model B this means that $\ell_A - \frac{3}{2}\ln(n) - [\ell_B - \frac{1}{2}\ln(n)] > 0$,
which can be rewritten as $\ell_A - \ell_B > ln(n)$.

The likelihood ratio test has test statistic $2(\ell_A - \ell_B)$. Since model A has 3 parameters and model B has 1 parameter, the number of degrees of freedom in the chi-square statistic is $3 - 1 = 2$. The critical value for a test with significance level 5% is $\chi^2_{.05}(2) = 5.991$
and the critical value for a test with significance level 1% is $\chi^2_{.01}(2) = 9.210$.
Since the null hypothesis is not rejected at the 1% level, it must be true that $2(\ell_A - \ell_B) < 9.21$,
so that $\ell_A - \ell_B < 4.605$. From the Schwartz Bayesian Criterion, we had $ln(n) < \ell_A - \ell_B$,
and therefore, $ln(n) < 4.605$. It then follows that $n < e^{4.605} = 99.98$.
The maximum (integer) value for n is 99.