

# EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2005

## Week of December 5

A late-night television infomercial claims that the "Extend Your Life" rejuvenation cocktail has the following benefits for an 80-year old male:

- (i) doubles the probability of surviving 10 years, and
- (ii) increases expected future lifetime by 7.5 years.

A late-night actuary viewing this infomercial models survival from birth based on DeMoivre's Law with different  $\omega$  for before and after the rejuvenation cocktail is taken. Find the value of  $\omega$  for survival after the rejuvenation cocktail is taken.

**The solution can be found below.**

## Week of December 5 - Solution

Suppose that the DeMoivre age limit before the cocktail is taken is  $\omega$ , and after the cocktail is taken it is  $\omega'$ .

Before rejuvenation cocktail:  ${}_{10}p_{80} = \frac{\omega-90}{\omega-80}$  ,  ${}^e_{80} = \frac{\omega-80}{2}$  .

After rejuvenation cocktail:  ${}_{10}p'_{80} = \frac{\omega'-90}{\omega'-80}$  ,  ${}^e'_{80} = \frac{\omega'-80}{2}$  .

We are given that  $\frac{\omega'-90}{\omega'-80} = 2 \cdot \frac{\omega-90}{\omega-80}$  and  $\frac{\omega'-80}{2} = \frac{\omega-80}{2} + 7.5$  .

From the final equation, we get  $\omega' = \omega + 15$  , and then the previous equation can be written as

$$\frac{\omega+15-90}{\omega+15-80} = 2 \cdot \frac{\omega-90}{\omega-80} .$$

This equation can be written in the form  $(\omega - 75)(\omega - 80) = 2(\omega - 90)(\omega - 65)$  ,

which becomes the quadratic equation  $\omega^2 - 155\omega + 5700 = 0$  .

There are two roots,  $\omega = 60$  and  $\omega = 95$  . We discard  $\omega = 60$  as infeasible (since the model considers someone who is still alive at age 80).

Then  $\omega' = 110$  .