EXAM M QUESTIONS OF THE WEEK

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Week of December 26

An insurer issues a continuous 10-year certain and life annuity of 1 per year 100 independent lives all at age x. The insurer charges a single contract premium that is 1% larger than the single benefit premium. The mortality model used has constant force of mortality of .01 at all ages, and force of interest .05. Using the normal approximation, find the probability that the insurer's aggregate issue date loss for all 100 policies is greater than 0.

The solution can be found below.

Week of December 26 - Solution

The single benefit premium for the annuity is

$$\overline{a}_{\overline{10|}} + {}_{10|}\overline{a}_x = \frac{1-e^{-10\delta}}{\delta} + v^{10}\,{}_{10}p_x \cdot \overline{a}_{x+10} = \frac{1-e^{-10\delta}}{\delta} + e^{-10\delta}e^{-10\mu} \cdot \frac{1}{\delta+\mu}$$

$$= \frac{1-e^{-.5}}{.05} + e^{-.5}e^{-.1} \cdot \frac{1}{.05+.01} = 17.0162\ .$$

The contract premium charged for each annuity is $1.01 \times 17.0162 = 17.1864$.

Since the present value random variable of the 10-year certain and life annuity differs from a 10-year deferred annuity only by the constant $\overline{a}_{\overline{10|}}$, it follows that the variance of the present value random variable for the 10-year certain and life annuity is the same as the variance of a 10-year deferred annuity. The variance of the present value random variable of an n-year deferred continuous annuity of 1 per year is $Var[Y] = \frac{2}{\delta}v^{2n}\,{}_np_x(\overline{a}_{x+n} - {}^2\overline{a}_{x+n}) - ({}_{n|}\overline{a}_x)^2$.

For constant force of mortality μ, we have ${}^2\overline{a}_{x+n} = \frac{1}{2\delta+\mu}$.

For this 10-year deferred annuity, this is $Var[Y] =$

$$\frac{2}{.05} \cdot e^{-20(.05)} \cdot e^{-10(.01)}\left(\frac{1}{.05+.01} - \frac{1}{.10+.01}\right) - \left[e^{-10(.05)}e^{-10(.01)} \cdot \frac{1}{.05+.01}\right]^2 = 17.2050\ .$$

The issue date loss random variable for a single annuity is $L = Y - 18.7178$, with an expected value of $E[L] = E[Y] - 17.1864 = -.1702$, and a variance of $Var[L] = Var[Y] = 17.2050$.

The aggregate issue date loss for 100 independent annuities is $W = L_1 + \cdots + L_{100}$, with mean $E[W] = 100 \times (-.1702) = -17.02$ and with variance $Var[W] = 100 \times 17.2050 = 1,720.50$.

Using the normal approximation, the probability that the aggregate issue date loss is greater than 0 is $P[W > 0] = P\left[\frac{W-(-17.02)}{\sqrt{1,720.50}} > \frac{0-(-17.02)}{\sqrt{1,720.50}}\right] = 1 - \Phi\left(\frac{17.02}{\sqrt{1,720.50}}\right) = 1 - \Phi(.41) = .34$.