EXAM P QUESTIONS OF THE WEEK

S. Broverman, 2005

Week of December 19/05

A poll is taken in the northern Canadian riding of Moose and Squirrel Flats (voting districts in Canada are called "ridings"). According to the poll, the voter preferences in that riding are

Political Party	Poll Percentage
Liberal	36%
Conservative	32%
New Democrat	18%
Bloc Quebecois	8%
Green	4%
Natural Law	1%
Rhinoceros	1%

A group of 5 voters who live in Moose and Squirrel Flats is randomly chosen. Assume that the poll percentages are representative of the voting probabilities of each the 5 randomly chosen individuals. Find the probability that the votes of the 5 individuals result in at least 1 New Democrat vote and at most 2 Liberal votes and at most 2 Conservative votes.

(Historical Note: The Rhinoceros Party of Canada was a nationally registered political party that existed from the 1960's to the 1990's. One of the main positions of the party was a "promise to keep none of our promises.")

The solution can be found below.

Week of December 19 - Solution

We use the binomial and the multinomial distribution to find probabilities. The multinomial distribution has parameters $n,\ p_1,\ p_2, \ldots,\ p_k$ (where n is a positive integer and $0 \le p_i \le 1$ for all $i = 1, 2, ..., k$ and $p_1 + p_2 + \cdots + p_k = 1$). The interpretation of this distribution is as follows. Suppose that an experiment has k possible outcomes, with probabilities $p_1,\ p_2, ..., p_k$ respectively. If the experiment is performed n successive times (independently), let X_i denote the number of experiments that resulted in outcome i, so that $X_1 + X_2 + \cdots + X_k = n$.

The multinomial distribution probability function is

$P[X_1 = x_1, X_2 = x_2, \cdots, X_k = x_k] = p(x_1, x_2, ..., x_k) = \frac{n!}{x_1! \cdot x_2! \cdots x_k!} \cdot p_1^{x_1} \cdot p_2^{x_2} \cdots p_k^{x_k}$.

In this problem, an "experiment" corresponds to the vote of a randomly chosen voter. There are $k = 7$ possible outcomes for a vote (the 7 parties). Let X_L , X_C and X_N denote the number of Liberal, Conservative, and New Democrat votes, respectively, of the 5 voters. We wish to find $P[[X_L \le 2 \cap X_C \le 2 \cap X_N \ge 1]$. We consider all the possible combinations that result in this event.

If there are 5, 4 or 3 New Democrat votes ($X_N = 3, 4$ or 5), then there cannot be more than 2 Liberal or Conservative votes. The probability of this occurring is

$\binom{5}{5}(.18)^5 + \binom{5}{4}(.18)^4(.82) + \binom{5}{3}(.18)^3(.82)^2 = .0437$ (each is a binomial probability).

The probability of 2 New Democrat votes is $\binom{5}{2}(.18)^2(.82)^3 = .1786$.

The probability of 2 New Democrat votes and 3 Liberal votes is $\frac{5!}{2! \cdot 3!} \cdot (.18)^2(.36)^3 = .0151$, and the probability of 2 New Democrat votes and 3 Conservative votes is $\frac{5!}{3! \cdot 2!} \cdot (.18)^2(.32)^3 = .0106$ (these are multinomial probabilities).

Therefore, the probability of 2 New Democrat votes and at most 2 Liberal votes and at most 2 Conservative votes is $.1786 - .0151 - .0106 = .1529$.

The probability of 1 New Democrat vote is $\binom{5}{1}(.18)(.82)^4 = .4069$.
The probability of 1 New Democrat vote and 4 Liberal votes is $\frac{5!}{1!\cdot 4!} \cdot (.18)(.36)^4 = .0151$.
The probability of 1 New Democrat vote and 3 Liberal votes is
$\frac{5!}{1!\cdot 3!\cdot 1!} \cdot (.18)(.36)^3(.46) = .0773$.
The probability of 1 New Democrat vote and 4 Conservative votes is $\frac{5!}{1!\cdot 4!} \cdot (.18)(.32)^4 = .0094$.
The probability of 1 New Democrat vote and 3 Conservative votes is
$\frac{5!}{1!\cdot 3!\cdot 1!} \cdot (.18)(.32)^3(.50) = .0590$.
Therefore, the probability of 1 New Democrat votes and at most 2 Liberal votes and at most 2 Conservative votes is $.4069 - .0151 - .0773 - .0094 - .0590 = .2461$.

The overall probability that the votes of the 5 individuals result in at least 1 New Democrat vote and at most 2 Liberal votes and at most 2 Conservative votes is $.0437 + .1529 + .2461 = .4427$.