EXAM M QUESTIONS OF THE WEEK

S. Broverman, 2005

Week of December 19

Actuary A uses the following mortality model is used for life insurance valuation:

$$\mu^A(x) = \begin{cases} .01 & 0 < x \le 80 \\ \frac{1}{100-x} & 80 < x < 100 \end{cases} .$$

Actuary B uses the following mortality model is used for life insurance valuation:

$$\mu^B(x) = \begin{cases} .01 & 0 < x \le 60 \\ \frac{1}{100-x} & 60 < x < 100 \end{cases} .$$

The force of interest is .06. Each actuary calculates the single benefit premium for a continuous whole life insurance of 1 at age 50. Find $\overline{A}_{50}^{(A)} - \overline{A}_{50}^{(B)}$.

The solution can be found below.

Week of December 19 - Solution

We use the rule $\overline{A}_z = \overline{A}_{\overset{1}{z}:\overline{n}|} + {}_{n|}\overline{A}_z = \overline{A}_{\overset{1}{z}:\overline{n}|} + v^{-n} \cdot {}_np_z \overline{A}_{z+n}$.

With constant force of mortality μ and constant force of interest δ,
${}_np_x = e^{-\mu n}$, $\overline{A}_{\overset{1}{x}:\overline{n}|} = \frac{\mu(1-e^{-(\mu+\delta)n})}{\mu+\delta}$, and $\overline{A}_y = \frac{\mu}{\mu+\delta}$.
With force of mortality $\frac{1}{100-x}$, mortality follows DeMoivre's Law with $\omega = 100$, and
$\overline{A}_y = \frac{1}{100-y} \cdot \overline{a}_{\overline{100-y}|}$.

For actuary A, we have $\overline{A}_{50}^{(A)} = \overline{A}^{(A)}_{\overset{1}{50}:\overline{30}|} + {}_{30|}\overline{A}_{50}^{(A)} = \overline{A}^{(A)}_{\overset{1}{50}:\overline{30}|} + e^{-30\delta} \cdot {}_{30}p_{50}\overline{A}_{80}^{(A)}$.
Therefore,
$\overline{A}_{50}^{(A)} = \frac{(.01)(1-e^{-(.01+.06)(30)})}{.01+.06} + e^{-30(.06)} \cdot e^{-30(.01)} \cdot \frac{1}{100-80} \cdot \overline{a}_{\overline{100-80}|}$
$= .1967$.

For actuary B, we have $\overline{A}_{50}^{(B)} = \overline{A}^{(B)}_{\overset{1}{50}:\overline{10}|} + {}_{10|}\overline{A}_{50}^{(B)} = \overline{A}^{(B)}_{\overset{1}{50}:\overline{10}|} + e^{-10\delta} \cdot {}_{10}p_{50}\overline{A}_{60}^{(B)}$.
Therefore,
$\overline{A}_{50}^{(B)} = \frac{(.01)(1-e^{-(.01+.06)(10)})}{.01+.06} + e^{-10(.06)} \cdot e^{-10(.01)} \cdot \frac{1}{100-60} \cdot \overline{a}_{\overline{100-60}|}$
$= .2601$.

The absolute difference is .0634 .