

EXAM FM QUESTIONS OF THE WEEK

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Week of December 19

Jones is agrees to invest in a savings plan that requires deposits of \$1000 at the start of each year for 5 years. According to the terms of the savings plan, the force of interest at time t is $.02 + .002t^2$, and the first deposit occurs at time 0, with time measured in years). Find the level annual effective rate of interest that would result in the same accumulated value at the end of 5 years as he will have in his savings plan.

The solution can be found below.

Week of December 19 - Solution

The accumulation factor from time s_1 to s_2 is $e^{\int_{s_1}^{s_2} (.02 + .002t^2) dt} = e^{.02(s_2 - s_1) + \frac{.002}{3}(s_2^3 - s_1^3)}$.

The first deposit accumulates from time 0 to time 5, so the accumulated value is

$$1000e^{.02(5-0) + \frac{.002}{3}(125-0)} = 1201.21.$$

The second deposit accumulates from time 1 to time 5, so the accumulated value is

$$1000e^{.02(5-1) + \frac{.002}{3}(125-1)} = 1176.64.$$

The 3rd, 4th and 5th deposits accumulate to

$$1000e^{.02(5-2) + \frac{.002}{3}(125-8)} = 1147.98, 1000e^{.02(5-3) + \frac{.002}{3}(125-27)} = 1111.08,$$

$$\text{and } 1000e^{.02(5-4) + \frac{.002}{3}(125-64)} = 1062.54.$$

The total accumulate value of Smith's savings plan at the end of 5 years is 5699.45.

The equivalent annual effective rate i must satisfy $1000\ddot{s}_{\overline{5}|i} = 5699.45$.

Using the calculator unknown interest function, we get $i = .0440$.