EXAM C QUESTIONS OF THE WEEK

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Week of December 19

A study of the time until failure, X, of an electronic device is based on observing 20 of the devices. One failure and one right-censoring is observed at each of the integer time points 1, 2, . . , 10. The probability $P[3 < X < 6 \mid X \leq 9]$ is to be estimated.
Find the absolute difference between the Kaplan-Meier product limit estimate and the Nelson-Aalen estimate.

Solution can be found below.

Week of December 19 - Solution

$P[3 < X < 6 \,|\, X \leq 9] = \frac{P[3<X<6\,]}{P[X\leq 9]}$

Since the observed failure times are all integers, the estimate of $P[3 < X < 6\,]$ is the same as the estimate of $P[3 < X \leq 5\,] = S(3) - S(5)$.

Therefore, we wish to estimate $\frac{S(3)-S(5)}{1-S(9)}$.

The numbers at risk and the numbers of failures at each failure time are:

y_i :	1	2	3	4	5	6	7	8	9	10
r_i :	20	18	16	14	12	10	8	6	4	2
s_i :	1	1	1	1	1	1	1	1	1	1

The product limit estimate of $S(3)$ is $(1-\frac{1}{20})(1-\frac{1}{18})(1-\frac{1}{16}) = .841146$.

The product limit estimate of $S(5)$ is $(1-\frac{1}{20})(1-\frac{1}{18})(1-\frac{1}{16})(1-\frac{1}{14})(1-\frac{1}{12}) = .715975$.

The product limit estimate of $S(9)$ is

$(1-\frac{1}{20})(1-\frac{1}{18})(1-\frac{1}{16})(1-\frac{1}{14})(1-\frac{1}{12})(1-\frac{1}{10})(1-\frac{1}{8})(1-\frac{1}{6})(1-\frac{1}{4}) = .352394$.

The product limit estimate of $\frac{S(3)-S(5)}{1-S(9)}$ is $\frac{.841146-.715975}{1-.352394} = .1933$.

The Nelson-Aalen estimate of $H(3)$ is $\widehat{H}(3) = \frac{1}{20} + \frac{1}{18} + \frac{1}{16} = .168056$,
so the N-A esitmate of $S(3)$ is $e^{-.168056} = .845306$.

The Nelson-Aalen estimate of $H(5)$ is $\widehat{H}(5) = \frac{1}{20} + \frac{1}{18} + \frac{1}{16} + \frac{1}{14} + \frac{1}{12} = .322817$,
so the N-A esitmate of $S(3)$ is $e^{-.322817} = .724106$.

The Nelson-Aalen estimate of $H(9)$ is
$\widehat{H}(9) = \frac{1}{20} + \frac{1}{18} + \frac{1}{16} + \frac{1}{14} + \frac{1}{12} + \frac{1}{10} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4} = .964484$,
so the N-A estimate of $S(3)$ is $e^{-.964484} = .381180$.

The Nelson-Aalen estimate of $\frac{S(3)-S(5)}{1-S(9)}$ is $\frac{.845306-.724106}{1-.381180} = .1959$.

The absolute difference between the two estimates is .0026 .