EXAM P QUESTIONS OF THE WEEK

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A study is done of people who have been charged by police on a drug-related crime in a large urban area. A conviction must take place in order for there to be a sentence of jail time.
The following information is determined:
(a) 75% are convicted.
(b) 10% of those convicted actually did not commit the crime.
(c) 25% of those not convicted actually did commit the crime.
(d) 2% of those who actually did not commit the crime are jailed.
(e) 20% of those who actually did commit the crime are not jailed.
Find the probability that someone charged with a drug-related crime who is convicted but not sentenced to jail time actually did not commit the crime.

The solution can be found below.

Week of December 12 - Solution

Our "probability space" consists of all people who have been charged with a drug-related offence. We define the following events:
C - the person is convicted
T - the person is sentenced to jail time
D - the person did actually commit the crime.
Since jail time is sentenced only to those who are convicted, we have
$T \subset C$, so that $P(T \cap C) = P(T)$.
We are also given the following information: $T \subset C$, so that $P(T \cap C) = P(T)$,
$P(C) = .75$, $P(D'|C) = .10$, $P(D|C') = .25$, $P(T|D') = .02$, $P(T'|D) = .20$.

We wish to find $P(D'|C \cap T')$.

From the definition of conditional probability, we have $P(D'|C \cap T') = \frac{P(D' \cap C \cap T')}{P(C \cap T')}$.

From the given information, we get
$P(D' \cap C) = P(D'|C) \cdot P(C) = (.1)(.75) = .075$,
$P(D \cap C') = P(D|C') \cdot P(C') = (.25)(.25) = .0625$.
Since, $P(C) = .75$ and $P(C') = .25$, we get
$P(D \cap C) = P(C) - P(D' \cap C) = .75 - .075 = .675$.
Then $P(D) = P(D \cap C) + P(D \cap C') = .675 + .0625 = .7375$, and $P(D') = .2625$.

Then $P(T \cap D') = P(T|D') \cdot P(D') = (.02)(.2625) = .00525$ and
$P(T' \cap D) = P(T'|D) \cdot P(D) = (.20)(.7375) = .1475$.
Then $P(T \cap D) = P(D) - P(T' \cap D) = .7375 - .1475 = .59$,
and $P(T) = P(T \cap D) + P(T \cap D') = .59 + .00525 = .59525$, and $P(T') = .40475$.

Then $P(C \cap T') = P(C) - P(C \cap T) = P(C) - P(T) = .75 - .59525 = .15475$,
and
$P(D' \cap C \cap T') = P(D' \cap C) - P(D' \cap C \cap T)$
$= P(D' \cap C) - P(D' \cap T) = .075 - .00525 = .06975$.

Finally, $\frac{P(D' \cap C \cap T')}{P(C \cap T')} = \frac{.06975}{.15475} = .4507$.